

Prediction of Compressive Strength of Segmental Interlocking Stones Using Ibearugbulem's Regression Function

S. Sule¹ and C. Nwaobakata²

^{1,2} Department of Civil and Environmental Engineering, University of Port Harcourt, P.M.B 5323, Choba, Rivers State, Nigeria

ABSTRACT

In this paper, a mathematical model is developed to predict the compressive strength of segmental interlocking stones using Ibearugbulem's regression function. The formulated model was tested for adequacy at a 95% confidence level using Student's t-test and was adequate. The experimental points were also found to align with the predicted values at different points of observation, showing that the model formulated is reliable. A computer program developed using Visual Basic 6.0 was invoked to quickly predict the mix ratios corresponding to a specified value of compressive strength and vice versa.

Keywords: Compressive strength, segmental interlocking stones, regression function, computer program, mix ratios

INTRODUCTION

Cement is the most important and the most expensive component of concrete. Therefore the selection of concrete mix ratios must be made in such a way as to maximize the compressive strength of hardened concrete at the lowest possible cost [1-3]. Concrete is a versatile structural material in the world. Neville and Brook [4] defined concrete as a product of water, cement, and aggregates and which, when sufficiently hardened, is used to support various structural loads. The strength of concrete depends mostly on the relative proportion of the component materials [5-7].

Segmental interlocking stones are common highway pavement construction materials. Their durability is a function of the proportions of the component materials. The component materials are water, cement, and coarse sand. The mixture of these materials then forms a stone-like material called concrete. The end products may have different geometries and thicknesses.

The failure of most segmental interlocking stones used as pavement materials in Nigeria can be traced to a lack of knowledge of the mix proportions that would maximize their compressive strength characteristics. Compressive strength is the only property used to judge the quality of concrete [8]. The

durability of segmental pavement materials depends on its compressive strength.

According to Ibearugbulem et al. [1], Scheffe's and Osadebe's methods of concrete mixture optimization depend on a predetermined number of an experiment to develop the optimization models, and they can only make predictions for mix ratios that fall within the experimental domain [9-10].

In this paper, a mathematical model is developed based on Ibearugbulem's regression theory to predict the compressive strength of segmental interlocking stones and predict the mix ratios that would maximize the compressive strength of segmental interlocking stones as highway pavement construction materials. A computer program coded in the basic language is also developed to predict the best mix ratios quickly.

Ibearugbulem's Optimization Theory

According to Ibearugbulem et al. [1], the polynomial equation for solving a three mixture optimization problem is given by:

$$F(z) = \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_{12} Z_1 Z_2 + \beta_{13} Z_1 Z_3 + \beta_{23} Z_2 Z_3 + \beta_{123} Z_1 Z_2 Z_3 \quad (1)$$

$$Z_i = \frac{s_i}{S} \quad (2)$$

Where: Z_i represents the fractional portions of the actual mix ratios S_i .

And

$$S = s_1 + s_2 + s_3 \quad (3)$$

Ibearugbulem et al. gave the coefficients of equation (1) as:

$$[F(z)*Z] = [CC]*[\beta] \quad (4)$$

Where: CC matrix represents a symmetric matrix

The column vector $F(z)*Z$ is obtained by multiplying the vector of the laboratory responses, $F(z)$ by the column vector of the components fractional portions.

In this study, a 3-component concrete mixture is considered. In line with the Ibearugbulem's regression model for a 3-component concrete mixture, the CC



matrix is a 7 x 7 symmetric matrix as shown in equation (5).

$$CC = \begin{bmatrix} SZ_1Z_1 & SZ_1Z_2 & SZ_1Z_3 & SZ_1Z_1Z_2 & SZ_1Z_1Z_3 & SZ_1Z_2Z_3 & SZ_1Z_1Z_2Z_3 \\ SZ_1Z_2 & SZ_2Z_2 & SZ_2Z_3 & SZ_1Z_2Z_2 & SZ_1Z_2Z_3 & SZ_2Z_2Z_3 & SZ_1Z_2Z_3Z_2 \\ SZ_1Z_3 & SZ_2Z_3 & SZ_3Z_3 & SZ_1Z_2Z_3 & SZ_1Z_3Z_3 & SZ_2Z_3Z_3 & SZ_1Z_2Z_3Z_3 \\ SZ_1Z_1Z_2 & SZ_1Z_2Z_2 & SZ_1Z_2Z_3 & SZ_1Z_1Z_2Z_2 & SZ_1Z_1Z_2Z_3 & SZ_1Z_2Z_2Z_3 & SZ_1Z_1Z_2Z_3Z_2 \\ SZ_1Z_1Z_3 & SZ_1Z_2Z_3 & SZ_1Z_3Z_3 & SZ_1Z_1Z_2Z_3 & SZ_1Z_1Z_3Z_3 & SZ_1Z_2Z_3Z_3 & SZ_1Z_1Z_2Z_3Z_3 \\ SZ_1Z_2Z_3 & SZ_2Z_2Z_3 & SZ_2Z_3Z_3 & SZ_1Z_2Z_2Z_3 & SZ_1Z_2Z_3Z_3 & SZ_2Z_2Z_3Z_3 & SZ_1Z_1Z_2Z_3Z_3 \\ SZ_1Z_1Z_2Z_3 & SZ_1Z_2Z_3Z_2 & SZ_1Z_2Z_3Z_3 & SZ_1Z_1Z_2Z_3Z_2 & SZ_1Z_1Z_2Z_3Z_3 & SZ_1Z_1Z_2Z_3Z_3 & SZ_1Z_1Z_2Z_3Z_3Z_3 \end{bmatrix} \quad (5)$$

Where: $S = \sum \Sigma$

III. MATERIALS AND METHODS

The cement used as the binding agent was Dangote cement, with properties conforming to the specifications of BS 12 [15]. The water used in this study was fresh and free of organic matter. The coarse sand used in this study was obtained from the Imo River in Port Harcourt, Rivers state. It was washed and sundried for 2 weeks for usage. The grading and properties were carried out according to the provisions of BS 812 [12]. Eight mix ratios were used to formulate the model, while the remaining four mix ratios were used as checkpoints for model validation. The actual mix ratios, S, and their corresponding fractional portions, Z, are shown in Table 1. The mix design of

the component materials was carried out, and the component materials were weighed, adequately mixed, and compacted into 200mm x 100mm x 700mm molds. They were cured in water for 28 days, after which they were demoulded and tested for strength in compression. The compressive strength value was calculated using the formula:

$$f_c = \frac{P}{A} \quad (6)$$

Where: P , A = load at failure in compression and cross-sectional area of segmental interlocking stone, respectively.

Table 1, Values for S and Z

S/N	S1	S2	S3	SUM	Z1	Z2	Z3	Z1Z2	Z1Z3	Z2Z3	Z1Z2Z3
TMX1	0.45	1	6	7.45	0.060403	0.134228	0.805369	0.008108	0.048646	0.108103	0.0065297
TMX2	0.5	1	5	6.5	0.076923	0.153846	0.769231	0.011834	0.059172	0.118343	0.0091033
TMX3	0.55	1	5.5	7.05	0.078014	0.141844	0.780142	0.011066	0.060862	0.110658	0.0086329
TMX4	0.475	1	5.5	6.975	0.0681	0.143369	0.78853	0.009763	0.053699	0.113051	0.0076988
TMX5	0.5	1	5.75	7.25	0.068966	0.137931	0.793103	0.009512	0.054697	0.109394	0.0075444
TMX6	0.525	1	5.25	6.775	0.077491	0.147601	0.774908	0.011438	0.060048	0.114378	0.0088632
TMX7	0.4625	1	5.75	7.2125	0.064125	0.138648	0.797227	0.008891	0.051122	0.110534	0.007088
TMX8	0.49	1	5.5	6.99	0.0701	0.143062	0.786838	0.010029	0.055157	0.112566	0.0078909
Control mixes for model validation											
CMX1	0.495	1	5.4	6.895	0.071791	0.145033	0.783176	0.010412	0.056225	0.113586	0.0081545
CMX2	0.525	1	5.625	7.15	0.073427	0.13986	0.786713	0.010269	0.057766	0.11003	0.0080791
CMX3	0.5025	1	5.7	7.2025	0.069767	0.138841	0.791392	0.009687	0.055213	0.109877	0.0076659
CMX4	0.485	1	5.375	6.86	0.0707	0.145773	0.783528	0.010306	0.055395	0.114217	0.0080751

Where: S_1 = actual proportion of water, S_2 = actual proportion of cement, S_3 = actual proportion of coarse sand

Using equation (5), the CC matrix and the corresponding inverse are shown in Tables 2 and 3 respectively.

Table 2, CC matrix

0.04007	0.080641	0.443404	0.005744	0.031465	0.063351	0.004507
0.08061	0.826689	0.897027	0.011546	0.063351	0.128022	0.009066
0.443404	0.897027	4.954918	0.063351	0.348587	0.705654	0.049778
0.005744	0.011546	0.063351	0.000824	0.004507	0.009066	0.000647
0.031465	0.06335	0.348587	0.004507	0.024709	0.049778	0.003538
0.063351	0.128022	0.705654	0.009066	0.049778	0.100655	0.00712
0.004507	0.009066	0.049778	0.000647	0.003538	0.00712	0.000507

Table 3, Inverse of CC matrix

-230999	0.597768	-5275.541	-163594	303596.9	25125.9774	308749.5
-10.647	1.506443	-0.251654	-10.6596	14.67507	-0.5400078	11.19631
-5275.69	0.035212	-1656.708	-67390.6	26887.55	11112.1692	-48126.7
-163578	-1.63418	-67390.06	-2916435	919876.2	466470.059	-1177613
303594	-0.41373	26887.278	919895.1	-604857	-180982.78	249939.3
25140.23	-1.98603	11112.514	466483.5	-181002	-71622.85	359128.6
308765.2	-2.84548	-48126.5	-1177581	249914.7	359144.395	-3302464

IV. RESULTS AND DISCUSSION

Using the values of Z_i from Table 1 and the compressive strength values obtained from the laboratory (Table 4), $[F(z)*Z]$ was obtained as follows:

$$\sum(Z_1 * F(Z)) = 4.5423;$$

$$\sum(Z_2 * F(Z)) = 9.1849;$$

$$\sum(Z_3 * F(Z)) = 50.6677;$$

$$\sum(Z_1 Z_2 * F(Z)) = 0.6496;$$

$$\sum(Z_1 Z_3 * F(Z)) = 3.57;$$

$$\sum(Z_2 Z_3 * F(Z)) = 7.2233;$$

$$\sum(Z_1 Z_2 Z_3 * F(Z)) = 0.5103$$

The results obtained at the control points are shown in Table5.

Table 4, Compressive strength test results [13]

S/N	TMX1	TMX2	TMX3	TMX4	TMX5	TMX6	TMX7	TMX8
Lab Values (N/mm^2)	7.456	8.154	7.978	8.528	8.077	7.496	7.941	8.765
Model Values (N/mm^2)	8.008	8.31	7.91	8.076	7.922	8.104	8.025	8.041

Table 5, Control Points [13]

S/N	CMX1	CMX2	CMX3	CMX4
Lab Values (N/mm^2)	7.246	7.817	7.333	8.65
Model Values (N/mm^2)	8.076	7.906	7.933	8.108

Optimization Model Development

$$\begin{aligned}\beta_1 &= 56.2974, \\ \beta_2 &= 0.0024, \\ \beta_3 &= 14.0539, \\ \beta_{12} &= 422.0876, \\ \beta_{13} &= -195.9813, \\ \beta_{23} &= -18.7144, \\ \beta_{23} &= 217.9682\end{aligned}$$

Substitution of the above coefficient values into equation (1) yields:

$$\begin{aligned}F(Z) &= 56.2974Z_1 + 0.0024Z_2 + 14.0539Z_3 \\ &+ 422.0876Z_1Z_2 - 195.9813Z_1Z_3 \\ &- 18.7144Z_2Z_3 + 217.9682Z_1Z_2Z_3\end{aligned}\quad (7)$$

Equation (7) is the mathematical model for predicting the compressive strength of segmental interlocking stones based on Ibearugbulem's polynomial function.

The formulated mathematical model is now tested for reliability against the control points presented in Table 5 using the Student t-test at a 95% level of confidence and was found to be reliable (see Table 6).

Table 6, Student t-test on compressive strength of segmental interlocking stones

Y_{EXPT}	Y_{PRED}		Y_{EXPT}	Y_{PRED}
7.456	8.008	Mean	7.953	8.035
8.154	8.31	Variance	0.263	0.013
7.978	7.91	Observations	12	12
8.528	8.076	Pearson Correlation	0.234	
8.077	7.922	Hypothesized Mean Difference	0	
7.496	8.104	df	11	
7.941	8.025	t Stat	-0.566	
8.765	8.041	P(T<=t) one-tail	0.291	
7.246	8.076	t Critical one-tail	1.796	
7.817	7.906	P(T<=t) two-tail	0.583	
7.333	7.933	t Critical two-tail	2.201	
8.65	8.108			

Discussion of Results

The model developed was tested for adequacy using Student t-test at a 5% level of significance. The results of the test are as shown in Table 4. From Table 4, it can be seen that the calculated value of t (-0.566) is less than the table value (2.201), showing that the lack-of-fit is insignificant at a 95% confidence limit; hence, the prediction model is reliable. Also, the experimental values align with the predicted results testifying to the adequacy of the prediction model. The predicted mix ratios obtained from the present model based on the written computer program are also in consonance with those of the earlier model [14]. The computer prints out without waste of time, the mix ratios corresponding to a desired compressive strength value and vice versa.

V. CONCLUSION

The mathematical model for the prediction of compressive strength of segmental interlocking stone has been developed based on Ibearugbulem's regression function and tested for reliability at a 95% confidence level using Student's t-test. The

experimental values are almost identical to the predicted values showing that the formulated model can predict the compressive strength of segmental interlocking stones. The written computer program can print out the mix ratios corresponding to the desired strength value and vice versa.

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Private Sub STARTMNU_Click()

Cls

Text1.Text = ""

WW = InputBox("CLICK OK. TO CONTINUE"); Cls

WW = InputBox("CLICK OK. TO CONTINUE"); Cls

WW = InputBox("CLICK OK. TO CONTINUE"); Cls

CT = 0: YMAX = 0: KK = 0

ReDim X(8), A(8, 8), Z(8), N(8), B(8, 8)

Rem *** Coefficients of regression model ***

A1 = 56.2974: A2 = 0.0024: A3 = 14.0539: A4 = 422.0876: A5 = -195.9813

A6 = -18.7144: A7 = 217.9682

Rem *** Decision for calculating mix ratios given desired strength or otherwise ***

10 QQ = InputBox("WHAT DO YOU WANT TO DO? TO CALCULATE MIX RATIOS GIVEN

DESIRED COMPRESSIVE STRENGTH OR CALCULATING COMPRESSIVE STRENGTH GIVEN MIX RATIO?", " IF COMPRESSIVE STRENGTH IS KNOWN TYPE 1 ", "Type 1 or 0 and CLICK OK.")

If QQ <> 1 And QQ <> 0 Then EE = InputBox("No Way! You must ENTER 1 or 0", "CLICK OK and do so"): GoTo 10

If QQ = 0 Then GoTo 100

Rem Put in the value of strength desired here

YY = InputBox("WHAT IS THE DESIRED COMPRESSIVE STRENGTH?"): YY = 1 * YY

Rem *** Here is where the Actual Strength is calculated ***

For Z1 = 0.0604 To 0.078 Step 0.0001

For Z2 = 0.134 To 0.153 Step 0.001

Z3 = 1 - Z1 - Z2

Rem *** The Predictors will be calculated here ***

Z4 = Z1 * Z2: Z5 = Z1 * Z3: Z6 = Z2 * Z3: Z7 = Z1 * Z2 * Z3

Rem CACCULATING ACTUAL STRENGTH

YACT = A1 * Z1 + A2 * Z2 + A3 * Z3 + A4 * Z4 + A5 * Z5

YACT = YACT + A6 * Z6 + A7 * Z7

Y = YACT

If Z1 / Z2 < 0.45 Then GoTo 30

If Z1 + Z2 + Z3 <> 1 Then GoTo 30 'Or Z1 + Z2 + Z3 < 1

If Y > YY - 0.05 And Y < YY + 0.05 Then GoTo 20 Else GoTo 30

20 Text1.Text = Text1.Text + CStr("Compressive Strength" & vbTab & Format(YACT, "0.00#") & ",") & vbTab

Text1.Text = Text1.Text + CStr(" WATER = " & vbTab & Format(Z1 / Z2, "0.00#") & ",") & vbTab

```
Text1.Text = Text1.Text + CStr("    CEMENT
=" & vbTab & Format(Z2 / Z2, "0.00#") & ",") &
vbTab
```

```
Text1.Text = Text1.Text + CStr("    SAND ="
& vbTab & Format(Z4 / Z2, "0.00#") & ",") & vbTab
```

```
30
```

```
Next Z2
```

```
Next Z1
```

```
70 'Print "Sorry! Desired strength is outside the range
of the model"
```

```
111 GoTo 222
```

```
100 Rem *** Here is where the input of the principal
predictors are made ***
```

```
Cls
```

```
Z1 = InputBox("What is Water/Cement ratio"): Z1
= Z1 * 1
```

```
Z2 = InputBox("What is Cement value"): Z2 = Z2 *
1
```

```
Z3 = InputBox("What is Sand value"): Z3 = Z3 * 1
```

```
Z2 = Z2
```

```
TZT = Z1 + Z2 + Z3
```

```
Z1 = Z1 / TZT: Z2 = Z2 / TZT: Z3 = Z3 / TZT
```

```
Rem *** The predictors are calculated here
***
```

```
Z4 = Z1 * Z2: Z5 = Z1 * Z3: Z6 = Z2 * Z3: Z7 =
Z1 * Z2 * Z3
```

```
Rem calculating actual strength
```

```
YACT = A1 * Z1 + A2 * Z2 + A3 * Z3 + A4 * Z4
+ A5 * Z5
```

```
YACT = YACT + A6 * Z6 + A7 * Z7
```

```
Text1.Text = Text1.Text + CStr("Compressive
Strength" & vbTab & Format(YACT, "0.00#") & ",")
& vbTab
```

```
Text1.Text = Text1.Text + CStr("    WATER
=" & vbTab & Format(Z1 / Z2, "0.00#") & ",") &
vbTab
```

```
Text1.Text = Text1.Text + CStr("    CEMENT
=" & vbTab & Format(Z2 / Z2, "0.00#") & ",") &
vbTab
```

```
Text1.Text = Text1.Text + CStr("    Sand =" &
vbTab & Format(Z4 / Z2, "0.00#") & ",") & vbCrLf
```

```
222
```

```
End Sub
```

```
Private Sub STOPMNU_Click()
```

```
End
```

```
End Sub
```

Desired Strength: 8.18 N/mm²

Compressive Strength	8.009,	WATER = 0.456,	CEMENT = 1.00,	SAND = 5.897,
Compressive Strength	8.03,	WATER = 0.453,	CEMENT = 1.00,	SAND = 5.847,
Compressive Strength	7.986,	WATER = 0.46,	CEMENT = 1.00,	SAND = 5.947,
Compressive Strength	8.007,	WATER = 0.457,	CEMENT = 1.00,	SAND = 5.896,
Compressive Strength	8.028,	WATER = 0.453,	CEMENT = 1.00,	SAND = 5.846,
Compressive Strength	8.049,	WATER = 0.45,	CEMENT = 1.00,	SAND = 5.796,
Compressive Strength	7.984,	WATER = 0.461,	CEMENT = 1.00,	SAND = 5.947,
Compressive Strength	8.005,	WATER = 0.457,	CEMENT = 1.00,	SAND = 5.896,