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Cost Optimization of Chikoko-Cement Concrete Using Scheffe's Polynomial Function

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Abstract
In this paper, Scheffe's simplex lattice design technique is used to optimize the overall cost of chikoko-cement concrete. The cost model was developed based on the unit costs of the chikoko-cement concrete ingredients. The predicted cost values were compared with the observed cost values and were found to be in consonance at all points of observation. The reliability of the model was investigated using F statistic at a 5% level of significance and was found to be adequate. A computer program written in BASIC language was invoked to select the optimum mix ratios corresponding to a given cost value and vice versa. The computer program requires less effort to execute.

Keywords - *Chikoko-cement concrete, Level of significance, Mix ratios, Observed Cost, Overall Cost, Predicted Cost, Scheffe's simplex lattice design technique.*

INTRODUCTION

The unprecedented increase in the price of cement in recent times has slowed down the pace of infrastructural developments in Nigeria. Consequently, most citizens of Nigeria find it difficult to afford their decent shelters [1-3]. Nigeria is blessed with abundant local building and construction materials, such as stones, sand, laterite, and timber. However, a good number of Nigerians are still faced with the problem of securing their decent shelters [4]. Durability wise, concrete has been rated as the number one of all structural materials across the globe [5].

The search for decent shelters at this period of economic recession in Nigeria is increasing by the day [6]. Consequently, the local demand for cement by both the regional and urban dwellers increases. The establishment of cement factories at every nook and cranny of Nigeria is no guarantee for an increase in the availability of cement [7]. According to Johnarry, the effort by the Federal Government of Nigeria to engage in cement importation has always been a failure due to the high cost associated with foreign exchange.

The search for cement replacement materials in concrete production now becomes a necessity as it seems to be the only way out of the present housing problem in Nigeria.

The use of chikoko clay as cement partial replacement material in concrete production has been studied by some researchers [8-9]. The addition of chikoko clay in concrete would increase the number of concrete ingredients and would eventually make it difficult to identify the optimum mix ratios of the component materials. Concrete structures and houses perform better in service when the component materials are blended in optimum proportions. The use of laboratory trial mixes is not encouraging, as it will always lead to the production of uneconomical concrete [10].

In this paper, a mathematical model is developed based on Scheffe's simplex lattice design approach to optimize the cost of chikoko-cement concrete. A computer program written in a BASIC language is invoked to quickly select the best mix ratios corresponding to the desired overall cost value and vice versa.

II. MATERIALS AND METHODS

The cement used as the binder is ordinary portland cement, with properties meeting the requirements of [11]. The water used was free from any form of organic matter. The fine aggregate used was sharp river sand obtained from Otamiri River at the Federal University of Technology, Owerri, Imo State. The maximum size was 5mm. The grading and properties were carried out according to the specifications of [12]. The granites used as coarse aggregates were obtained in bags at Ihiagwua in Imo State. They were washed properly and sundried to remove dirt before usage. The maximum size was 20mm. The chikoko clay was obtained in bags from the mangrove swamps of the Eagle's Island in Port Harcourt, Rivers State. It was sundried for three weeks, after which it was ground and sieved to obtain particles as fine as cement. The design matrix for trial and control points based on Scheffe's (5, 2) factor space are presented in tables 1 and 2, respectively. The chikoko clay - oxide composition test results are shown in Table 3.

Table 1, Design matrix for trial points based on Scheffe's (5, 2) factor space

Z_1	Z_2	Z_3	Z_4	Z_5	**Overall Cost**	X_1	X_2	X_3	X_4	X_5
0.52601	0.947	0.053	2.1	4.2	Y_1	1	0	0	0	0
0.566	0.91901	0.081	2.02	4.04	Y_2	0	1	0	0	0
0.589	0.823	0.17701	1.91	3.82	Y_3	0	0	1	0	0
0.611	0.889	0.111	2.1601	4.32	Y_4	0	0	0	1	0
0.596	0.846	0.154	2.15	4.301	Y_5	0	0	0	0	1
0.546005	0.933005	0.067	2.06	4.12	Y_{12}	0.50	0.50	0	0	0
0.557505	0.885	0.115005	2.005	4.01	Y_{13}	0.50	0	0.50	0	0
0.568505	0.918	0.082	2.13005	4.26	Y_{14}	0.50	0	0	0.50	0
0.561005	0.8965	0.1035	2.125	4.2505	Y_{15}	0.50	0	0	0	0.50
0.5775	0.871005	0.129005	1.965	3.93	Y_{23}	0	0.50	0.50	0	0
0.5885	0.904005	0.096	2.09005	4.18	Y_{24}	0	0.50	0	0.50	0
0.581	0.882505	0.1175	2.085	4.1705	Y_{25}	0	0.50	0	0	0.50
0.6	0.856	0.144005	2.03505	4.07	Y_{34}	0	0	0.50	0.50	0
0.5925	0.8345	0.165505	2.03	4.0605	Y_{35}	0	0	0.50	0	0.50
0.6035	0.8675	0.1325	2.15505	4.3105	Y_{45}	0	0	0	0.50	0.50

Table 2, Design matrix for control points based on Scheffe's (5, 2) factor space

Z_1	Z_2	Z_3	Z_4	Z_5	**Overall Cost**	X_1	X_2	X_3	X_4	X_5
0.560337	0.896337	0.10367	2.01	4.02	Y C_1	0.333333	0.333333	0.333333	0	0
0.575337	0.886333	0.11367	2.0567	4.113333333	Y C_2	0.333333	0	0.333333	0.333333	0
0.57767	0.894	0.106	2.1367	4.273666667	Y C_3	0.333333	0	0	0.333333	0.333333
0.573003	0.894503	0.105503	2.047525	4.095	Y C_4	0.25	0.25	0.25	0.25	0
0.580503	0.87625	0.123753	2.080025	4.16025	Y C_5	0.25	0	0.25	0.25	0.25
0.569253	0.883753	0.116253	2.045	4.09025	Y C_{12}	0.25	0.25	0.25	0	0.25
0.551755	0.909003	0.091003	2.0325	4.065	Y C_{13}	0.5	0.25	0.25	0	0
0.576753	0.8655	0.134503	2.0775	4.1555	Y C_{14}	0.25	0	0.25	0	0.5
0.563604	0.905002	0.095002	2.05802	4.116	Y C_{15}	0.4	0.2	0.2	0.2	0
0.577602	0.884802	0.115202	2.06802	4.1362	Y C_{23}	0.2	0.2	0.2	0.2	0.2
0.573603	0.887601	0.112402	2.07602	4.1522	Y C_{24}	0.3	0.1	0.2	0.2	0.2
0.586101	0.879002	0.121002	2.07403	4.1482	Y C_{25}	0.1	0.2	0.2	0.3	0.2
0.565254	0.886552	0.113453	2.053	4.10625	Y C_{34}	0.35	0.15	0.25	0	0.25
0.574453	0.891002	0.109002	2.07752	4.1552	Y C_{35}	0.25	0.2	0.15	0.2	0.2
0.566005	0.903701	0.0963	2.12302	4.2463	Y C_{45}	0.45	0.05	0	0.2	0.3

Where**:**

Z_1 = actual proportion of water; X_1 = pseudo proportion of water
Z_2 = actual proportion of cement; X_2 = pseudo proportion of cement
Z_3 = actual proportion of chikoko; X_3 = pseudo proportion of chikoko
Z_4 = actual proportion of sand; X_4 = pseudo proportion of sand
Z_5 = actual proportion of granite; X_5 = pseudo proportion of granite

III. DEVELOPMENT OF OPTIMIZATION MODEL

According to Okere et al. [13], the overall cost of a 5-component concrete mixture based on a (5, 2) factor space is given by a real-valued scalar function as:

$$Y = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4 + \alpha_5 X_5 + \alpha_{12} X_1 X_2 + \alpha_{13} X_1 X_3 + \alpha_{14} X_1 X_4 + \alpha_{15} X_1 X_5 + \alpha_{23} X_2 X_3 + \alpha_{24} X_2 X_4 + \alpha_{25} X_2 X_5 + \alpha_{34} X_3 X_4 + \alpha_{35} X_3 X_5 + \alpha_{45} X_4 X_5 \quad \textbf{(1)}$$

According to Scheffe [14], the model coefficients are obtained as follows:

$$\alpha_i = Y_i \; and \; \alpha_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \qquad (2)$$

Where:

$$\alpha_i = \alpha_1, \alpha_3, \ldots, \alpha_5; \quad \alpha_{ij} = \alpha_{12}, \alpha_{13}, \alpha_{14}, \ldots., \alpha_{45}$$

Equation (1) is subject to the constraint:

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1 \quad (3)$$

Where: $X_i \geq 0 =$ the components proportion, $q =$ number of components in the mixture.

According to Scheffe, the number of experiments required to formulate the model for a given factor space is given by:

$$N = \frac{(q+n-1)!}{n!(q-1)!} \quad (4)$$

Where: n, q = Degree of the polynomial and number of concrete components respectively

Let the actual and pseudo components be denoted by Z_i and X_i. The relationship between

Z_i and X_i is given by:

$$Z = AX \quad (5)$$

Where: A = the coefficient matrix, Z = actual mix ratios and X = pseudo mix ratios

Using equation (5), the actual mix ratios (Z_i) are obtained and are given in matrix form as:

$$A = \begin{pmatrix} 0.526 & 0.566 & 0.589 & 0.611 & 0.596 \\ 0.947 & 0.919 & 0.823 & 0.889 & 0.846 \\ 0.053 & 0.081 & 0.177 & 0.111 & 0.154 \\ 2.100 & 2.020 & 1.910 & 2.160 & 2.150 \\ 4.200 & 4.040 & 3.820 & 4.320 & 4.300 \end{pmatrix} \quad (6)$$

One cubic meter of chikoko-cement concrete weighs 2400 Kg. Therefore, the observed overall cost of chikoko-cement concrete per observation point is given by:

$$OC = \frac{2400}{TZT} * \begin{pmatrix} Z_1 * CW + Z_2 * CC + Z_3 * CCH \\ + Z_4 * CS + Z_5 * CG \end{pmatrix} \quad (7)$$

Where:

CCC, CW, CC, CCH, CS, CG = the overall cost of concrete, Cost of water, Cost of cement, Cost of chikoko, Cost of sand, and Cost of coarse aggregate, respectively.

$$TZT = Z_1 + Z_2 + Z_3 + Z_4 + Z_5 \quad (8)$$

Z_1, Z_2, Z_3, Z_4, Z_5 = The actual proportion of water, cement, chikoko, sand, and coarse aggregate, respectively

The unit cost of the component materials are as follows:

Water	=	₦1.20,
Chikoko	=	₦4,
Cement	=	₦46,
Sand	=	₦2.33, and
Granite	=	₦8.

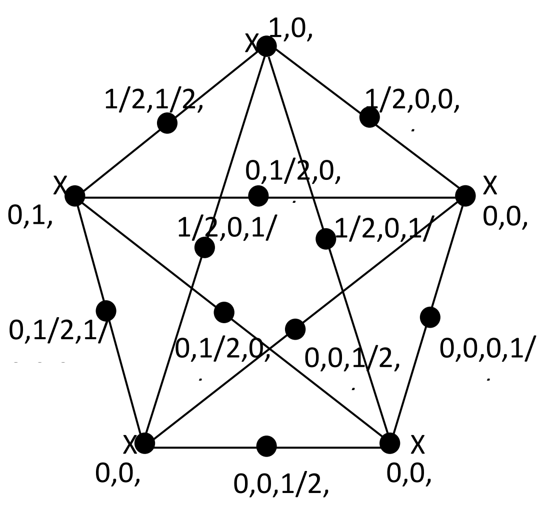

Fig. 1, A simplex lattice for a 5-component mixture used in this study

Table 3, Oxide composition of chikoko clay

CaO	SiO_2	$Al2O_3$	Fe_2O_3	MgO	Na_2O	K_2O	SO_3	TiO_2	ZnO	LoI
9.85	41.21	10.15	2.31	5.02	1.97	8.17	0.08	0.72	0.09	6.51

Source: Authors' experiment

A. Cost analysis of chikoko-cement concrete

The unit costs of the component materials are as follows:

Water =₦3.20, Cement =₦30.00, Sand = ₦1.60, Chikoko =₦4.00, and Granite =₦8.00.

The quantities and the overall cost of component materials per cubic meter of chikoko-cement concrete are given in Tables 4 and 5, respectively.

Table 4, Quantities of materials in kg per cubic meter of chikoko-cement concrete

S/No	Water	Cement	Chikoko	Sand	Granite
1.	161.311	290.416	16.253	644.006	1288.013
2.	178.127	289.224	25.492	635.719	1271.438
3.	193.141	269.873	58.044	626.314	1252.628
4.	181.236	263.697	32.925	640.734	1281.408
5.	177.756	252.318	45.930	641.233	1282.764
6.	169.610	289.828	20.813	639.916	1279.833
7.	176.693	280.488	36.449	635.456	1270.913
8.	171.440	276.834	24.728	642.343	1284.655
9.	169.648	271.102	31.298	642.600	1285.352
10.	185.480	279.747	41.433	631.113	1262.226
11.	179.728	276.083	29.318	638.301	1276.571
12.	177.936	270.275	35.985	638.550	1277.253
13.	186.890	266.630	44.855	633.885	1267.739
14.	185.084	260.679	51.700	634.127	1268.410
15.	179.501	258.023	39.410	640.983	1282.084
Control points					
16.	177.174	283.414	32.780	635.544	1271.089
17.	178.275	274.641	35.222	637.294	1274.567
18.	173.561	268.602	31.848	641.970	1284.020
19.	178.239	278.245	32.818	636.905	1273.794
20.	178.142	268.899	37.977	638.307	1276.676
21.	177.326	275.294	36.213	637.030	1274.137
22.	173.116	285.205	28.553	637.709	1275.417
23.	177.241	265.975	41.334	638.432	1277.018
24.	174.815	280.707	29.467	638.341	1276.670
25.	178.139	272.883	35.530	637.800	1275.649
26.	176.452	273.044	34.577	638.626	1277.301
27.	180.146	270.173	37.192	637.482	1275.007
28.	175.624	275.451	35.250	637.866	1275.809
29.	176.592	273.902	33.508	638.649	1277.348
30.	171.185	273.320	29.125	642.097	1284.272

Table 5, cost of materials per m^3 of chikoko-cement concrete

S/No	Water	Cement	Chikoko	Sand	Granite	Total Cost (₦)
1	516.1962	8,712.486	65.01397	1,030.41	10,304.1	20,628.21
2	570.0071	8,676.716	101.9668	1,017.151	10,171.51	20,537.35
3	618.0508	8,096.177	232.1757	1,002.103	10,021.03	19,969.53
4	579.9558	7,910.914	131.7003	1,025.174	10,251.26	19,899.01
5	568.8182	7,569.529	183.7206	1,025.972	10,262.11	19,610.15
6	542.7534	8,694.832	83.25125	1,023.866	10,238.66	20,583.36
7	565.4187	8,414.647	145.7968	1,016.73	10,167.3	20,309.9
8	548.6069	8,305.025	98.91243	1,027.748	10,277.24	20,257.54
9	542.8735	8,133.051	125.1936	1,028.16	10,282.81	20,112.09
10	593.5355	8,392.409	165.7339	1,009.781	10,097.81	20,259.27
11	575.1286	8,282.485	117.2735	1,021.281	10,212.56	20,208.73
12	569.3967	8,108.252	143.9417	1,021.68	10,218.02	20,061.3
13	598.0489	7,998.905	179.4209	1,014.216	10,141.91	19,932.5
14	592.2683	7,820.378	206.8003	1,014.603	10,147.28	19,781.33
15	574.4022	7,740.688	157.6394	1,025.572	10,256.67	19,754.97
Control points						
16	566.9556	8,502.416	131.1182	1,016.871	10,168.71	20,386.07
17	570.481	8,239.238	140.8882	1,019.67	10,196.54	20,166.82
18	555.3937	8,058.05	127.3905	1,027.152	10,272.16	20,040.15
19	570.364	8,347.344	131.2714	1,019.048	10,190.35	20,258.38
20	570.0534	8,066.969	151.9067	1,021.291	10,213.4	20,023.63
21	567.4421	8,258.828	144.854	1,019.247	10,193.1	20,183.47
22	553.9723	8,556.149	114.2109	1,020.334	10,203.34	20,448
23	567.1705	7,979.251	165.3354	1,021.492	10,216.15	19,949.39
24	559.4064	8,421.204	117.8681	1,021.346	10,213.36	20,333.19
25	570.044	8,186.478	142.1182	1,020.48	10,205.19	20,124.31
26	564.6462	8,191.322	138.3085	1,021.801	10,218.41	20,134.49
27	576.4681	8,105.203	148.7666	1,019.971	10,200.05	20,050.46
28	561.997	8,263.534	140.9991	1,020.585	10,206.47	20,193.59
29	565.0953	8,217.073	134.033	1,021.839	10,218.78	20,156.82
30	547.7933	8,199.597	116.5018	1,027.355	10,274.18	20,165.43

B. Cost Model Development

The overall cost values in Table 5 are substituted into equation (2) to yield the cost coefficients as follows:

$$\alpha_1 = 25422.370,\ \alpha_2 = 25272.750,$$
$$\alpha_3 = 24358.420,\ \alpha_4 = 24223.430,$$
$$\alpha_5 = 23759.820,\ \alpha_{12} = 3.88,\ \alpha_{13} = 71.22,$$
$$\alpha_{14} = -39.96,\ \alpha_{15} = -46.3,\ \alpha_{23} = 37.58,$$
$$\alpha_{24} = -62.12, \alpha_{25} = -81.3,\ \alpha_{34} = -13.54,$$
$$\alpha_{35} = -56.72,\ \alpha_{45} = 2.54$$

The obtained coefficients values are substituted into equation (1) to yield:

$$y = 2542237X_1 + 2527275X_2 + 24358420X_3 + 24223430X_4 + 23759820X_5 + 3.88X_1X_2 + 7.22X_1X_3 - 39.96X_1X_4 - 46.3X_1X_5 + 37.58\ 2_2X_3 - 62.12X_2X_4 - 81{,}3X_2X_5 - 13.54X_3X_4 - 56.72X_3X_5 + 2.54X_4X_5 \quad (9)$$

Equation (9) represents the mathematical model for the prediction of the overall cost of chikoko-cement concrete based on Scheffe's second-degree polynomial equation.

C. Test of reliability of the model

The model developed is tested for reliability against the control values at a 5% level of significance using F statistic, and the results are presented in Table 6.

Table 6, **F-statistic test of control points**

Control point	y_o	y_p	$y_o - \bar{y}_o$	$y_p - \bar{y}_p$	$(y_o - \bar{y}_o)^2$	$(y_p - \bar{y}_p)^2$
C_1	25,030.29	25,030.366	348.471	348.369	121432.270	121361.100
C_2	24,669.75	24,670.042	-12.069	-11.955	145.653	142.917
C_3	24,459.29	24,459.238	-222.529	-222.759	49519.007	49621.483
C_4	24,818.76	24,819.059	136.941	137.062	18752.929	18786.047
C_5	24,435.69	24,435.838	-246.129	-246.159	60579.321	60594.155
C_6	24,698.56	24,698.863	16.741	16.866	280.272	284.469
C_7	25,130.58	25,130.714	448.761	448.717	201386.734	201347.126
C_8	24,316.57	24,316.681	-365.249	-365.316	133406.589	133455.634
C_9	24,940.86	24,941.156	259.041	259.159	67102.412	67163.491
C_{10}	24,599.75	24,599.969	-82.069	-82.028	6735.266	6728.560
C_{11}	24,616.48	24,616.708	-65.339	-65.289	4269.141	4262.627
C_{12}	24,478.31	24,478.436	-203.509	-203.561	41415.777	41436.999
C_{13}	24,715.2	24,715.502	33.381	33.505	1114.313	1122.598
C_{14}	24,652.29	24,652.492	-29.529	-29.505	871.942	870.533
C_{15}	24,664.9	24,664.888	-16.919	-17.109	286.241	292.711
Σ	24,681.819	24,681.997			707297.869	707470.450

The F statistic is given by:

$$F = S_1^2 / S_2^2 \qquad (10)$$

S_1 , S_2 = larger and smaller value of the sample variances

Let: y_o , y_p = observed and predicted cost of chikoko-cement concrete

$$\bar{y}_o = \frac{\Sigma y_o}{n}; \quad \bar{y}_p = \frac{\Sigma y_p}{n}$$

Using equation (10), S_o^2 and S_p^2 are determined as follows:

$$S_o^2 = \frac{707297.869}{14} = 50521.2763$$

$$S_p^2 = \frac{707470.450}{14} = 50533.6035$$

Using equation (9), F is obtained as:

$$F = \frac{50533.6035}{50521.2763} = 1.000244$$

From Fisher table, $F_{0.95}(14,14) = 2.4$

The calculated value of F is less than the critical value (2.4) of F obtained from the standard statistical table. The prediction model is, therefore, adequate.

IV. CONCLUSION

This study developed a regression model to predict the cost of chikoko-cement concrete using Scheffe's simplex lattice design technique. The results obtained from F-statistic at a 5% level of significance showed that the cost model is adequate.

The predicted cost and observed values are in consonance at all points of observation, testifying to the reliability of the model. The overall cost of chikoko-cement concrete varied with the proportion of the component materials: water, cement, chikoko sand, and coarse aggregate, respectively.

With the computer program, any desired value of the overall cost that falls within the domain of the observed overall cost values can be easily predicted and vice versa.

REFERENCES

[1] O.S. Obam, and N.N. Osadebe, "*Optimization of compressive strength of rice husk ash pozzolan concrete". *Journal of Scientific and Industrial Studies, 1(2), 2006, 51-57.

[2] N.N. Osadebe, and S.O. Obam, *"Improvements in the properties of concrete by partial substitution of ordinary portland cement rice husk ash, Proceedings of Annual Conference of IRDI Science and Technology Forum"*, University of Nigeria Nsukka. 2(2), 2006, 87-91.

[3] E.E. Ndububa, and N.N. Osadebe, *"An optimization of the flexural strength of fibre cement mixture using Scheffe's Simplex Lattice".* NSE Technical Transaction, 42, (1), 2007, 1-17

[4] D.O. Onwuka, C.E. Okere, and S.U. Onwuka, *"prediction of concrete mix cost using modified regression theory"*, Nigerian Journal of Technology, Vol. 32, No.2, 2013, 211-216.

[5] A.M. Neville, and J.J. Brook, *"Concrete technology"*, Longman Ltd., Singapore, 1990.

[6] THISDAY Newspaper, *"Economic situation affecting the health of Nigerians"*, October 2016.

[7] T. Johnarry, and E.H. Alabo, Processed silty clays of alluvial mud-flats as pozzolan/filter.*"The proceedings of the national conference on construction engineering and management, organized by Department of Civil Engineering"*, University of Port Harcourt, Rivers state, 1987, 393-402.

[8] G.R. Otoko, *"On the economic use of cement in soil stabilization".* International Journal of Engineering and Technology Research, Vol.2, No.1, 2014, 01-07.

[9] D.O. Onwuka, D.O. and S. "Sule, *"Prediction of compressive strength of chikoko-cement concrete using Scheffe's" Polynomial function".* USEP: Journal of Research Information in Civil Engineering, Vol. 14, No 1, 2017, 1338-1358.

[10] L. Anyaogu, and J.C. Ezeh, *"Optimization of compressive strength of fly ash blended cement concrete using Scheffe's Simplex Theory".* Natural and Applied Sciences, Vol. 4, No.2, 2013, 177-186.

[11] BS 12: *"Specification for Portland cement. British Standards Institution"*, 1978.

[12] BS 812: Part, *"Methods for determination of particle size and shape"*, 1975.

[13] C.E. Okere, D.O. Onwuka, S.U. Onwuka, and J.I. Arimanwa, *"Optimization of concrete mix cost using Scheffe's Simplest Lattice Theory, Journal of Innovative Research in Engineering and Sciences"*, 4(1), 2013, 443-454.

[14] H. Scheffe, *"Experiments with mixtures. Journal of Royal Statistical Society Series" B*, 20, 1958, 344 -360.