A New Model Based on Dimensional Analysis for Predicting Longitudinal Dispersion in Streams

Ikebude C.F[1], J.C. Agunwamba[2]
Department of Civil Engineering, University of Port Harcourt, Nigeria
Department of Civil Engineering, University of Nigeria, Nsukka.

Abstract

This paper presents a new model for predicting dispersion coefficient in a stream. The developed model is based on dimensional analysis. Application of dimensional analysis to water quality modelling is presented, pointing out possibilities of applying this methodology in water quality research. Buckingham pie theory method was used in this study which lead to the formation of four (4) dimensionless groups. Appropriate regression method was used to obtain the constants. Stream and air temperature was incorporated into the model alongside other conventional parameters proposed in literature that influences dispersion such as velocity, shear velocity, width and depth. The model was developed calibrated and evaluated using data from the new-calabar river, southern Nigeria. The model was compared with other existing conventional model for predicting dispersion coefficient and it performed satisfactorily. The developed model gave coefficient of determination value of 0.9716, root mean square error of 0.5721 and discrepancy ratio of 4% (0.04). These values are pointers to the fact that the model is reliable for prediction. Inclusion of temperature into the model improved the predictive capacity of the model.

Keywords: *Dimensional analysis, water quality, temperature, prediction, regression.*

I. INTRODUCTION

Stream pollution is a subject growing public concern in the world at large. There are a lot of environmental challenges caused by the increase in pollutant discharged into natural bodies due to the growth in municipal, industrial and agricultural activities. All these activities contribute significantly to stream pollution problems in natural water bodies. It is therefore necessary to develop models that can simulate their dispersion effectively which leads to proper water management. The development and application of computer-operated mathematical models to stimulate the movement of pollutant in rivers and thus to anticipate environmental problems has been the subject of extensive research by government, agencies, universities for many years. The problem of stream pollution in many situations, has an impact on the ecology and environment and may cause potential risk on the human health and local economy. Efficient and reliable estimates of damages on the water quality due to pollution could play essential role in establishing control strategy for environmental protection. Introduction and utilization of such measures are impossible without knowledge of various processes such as formation of water flow and transport of pollutant. Mathematical models could be very helpful to understand the dynamics of both water flow and pollutant transport. In this respect, mathematical modelling of water flows and the processes of transport diffusion of pollutant could play a major role in establishing scientifically, justified and practically reasonable programs for long term measures for a rational use of water resources.

The success of the mathematical model in solving practical problems depends on the convenience of the models and the quality of the software used for the simulation of the real processes. (James I.P., 2002) Prediction of the pollutant transport is providing valuable information for the management of river quality. The modelling is not usually an east task due to the complete chemical and biological process involved (Vasile et al., 2010). The objective of the present work is to develop a mathematical model based on dimension analysis for predicting longitudinal dispersion in stream.

II. MATERIAL AND METHOD

Study Area

The study area used to conduct the tracer dye experiment at different stream and surrounding air temperatures was a section of the new-Calabar river located at Aluu in Rivers state Nigeria. The new Calabar River is one of the most prominent rivers in the state; others include Bonny River, Andoni River, Nun River and Orashi River. New Calabar River is an acidic fresh and non-tidal river which took its rise from Elele-Alimini and empties into some creeks and lagoon bordering the Atlantic Ocean. The section of the river where the study was done is located at a geographical co- ordinate between latitude N4^{0}55.353^1- N4^{0}55.365^1 and longitude E006^0 53.785^1- Eoo6^0 53.793^1, at an elevation of 6m above the mean sea level. The new-Calabar river as served as a receiving water body for discharge of point and non-point wastes from companies that have their operations along the banks of the rivers, which has led to the pollution of the rivers over the years. New Calabar River is situated in the tropical rain forest which the climate is characterized by two seasons which are the raining season which commences between April to November and the dry season which commences between November to March. The people around the New-Calabar River are predominantly fisher men and they engage in a lot of dredging activities to earn their living. The New-Calabar River used for recreation sometimes and its fetched for domestic purpose and Agricultural purposes. The slope of the study area is generally a gentle sloping terrain as seen in Figure 1.

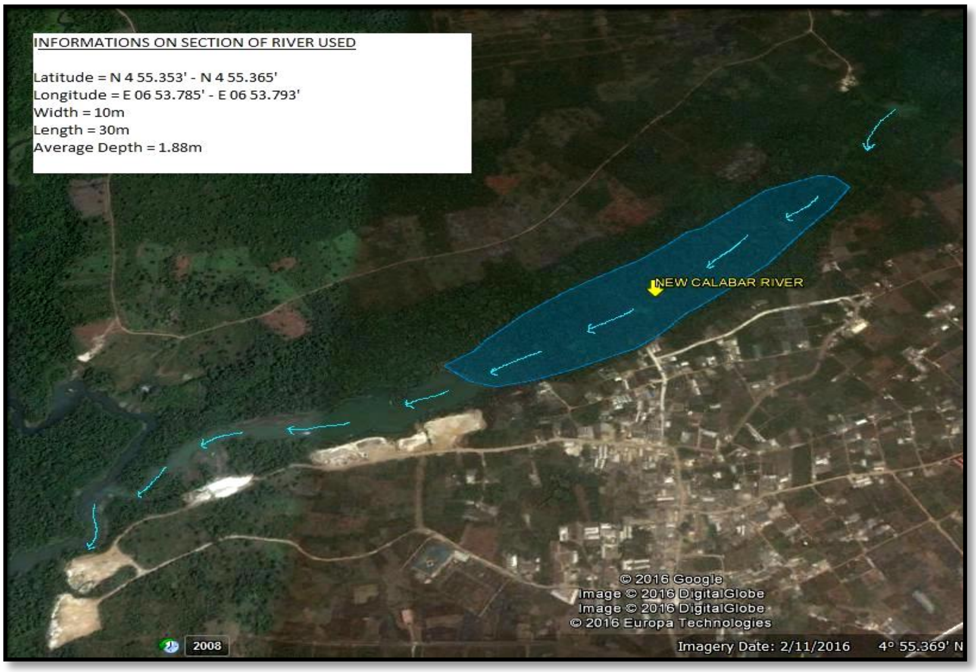

Fig. 1: Study Area Where Tracer Experiment Was Carried Out

The model describing dispersion in stream is based on the formation of a non-dimensional pie group from its stated variables influencing dispersion. The most important part of the model is the selection of appropriate variables. Many researchers have considered other hydraulic characteristic but this work seeks to introduce the impact of prevailing air and stream temperature as a key factor affecting dispersion. It is essential to state the parameter which characterizes the dispersion and which may be measured (Zelenakova and Scekova, 2006). The following parameters have been considered.

- Velocity (m/s)
- Shear velocity (m/s)
- Width (m)
- Depth (m)
- Temperature of river water (k)
- Temperature of air (k)
- Dispersion

All the given variables are presented in basic dimension which is the condition for dimensional analysis application.

The general relation among the selected variables which can affect pollutant/contaminant dispersion can be put down in the form below in order that each parameter is considered with the same dimension ϕ

$$\phi(D_L, V, U, W, D, T_R\ T_a) = 0 \qquad (1)$$

The dimensional matrix relation has the rank of matrix m= 3 and its lines are dimensionally independent from each other. From n=7 independent variable at matrix rank m, it is possible to set up n-m of non-dimensional pie groups.

- No of fundamental unit = 3
- No of variable = 7
- No of pie group = 7 – 3 = 4

Out of the seven variables, three (3) of the variables are selected as repeating variables based on the appropriate method of selection. The repeated variables selected are shear velocity, width, and temperature of river water.

$$\prod_1 = U_* \,.\, B \,.\, T_R \,.\, D_L \qquad (2)$$

$$\prod_2 = U_* \,.\, B \,.\, T_R \,.\, H \qquad (3)$$

$$\prod_3 = U_* \,.\, B \,.\, T_R \,.\, V \qquad (4)$$

$$\prod_4 = U_* \,.\, B \,.\, T_R \,.\, T_a \qquad (5)$$

Applying the principle of dimensional homogeneity and solving appropriately.

$$\prod_1 = \frac{D_L}{U_* B} \qquad (6)$$

$$\prod_2 = {}^{H}/_{B} \qquad (7)$$

$\prod_3 = \frac{V}{U_*}$ (8)

$\prod_4 = {T_a}/{T_s}$ (9)

$\prod_1 = f(\prod_2 \ \prod_3 \ \prod_4)$ (10)

$\frac{D_L}{U_* B} = f\left(\frac{H}{B_1}\ \frac{V}{U_*}\ \frac{T_a}{T_s}\right)$ (11)

$D_L = a.\ (U_* B)^b.\left(\frac{H}{B}\right)^c.\left(\frac{V}{U_*}\right)^c.\left(\frac{T_a}{T_R}\right)^e$ (12)

Equation (12) represents the model for predicting longitudinal dispersion in stream based on hydraulic parameter and temperature. The model is valid for any contaminant but it is necessary to obtain the regression coefficients (a-e). The regression coefficients were obtained using an add-in function in Microsoft excels. Prediction of longitudinal dispersion in a river was performed in the New-calabar river, Rivers State, Nigeria.

The hydraulic parameters and temperature of river water and surrounding air was monitored over a long period of time.

Table 1: Summarized Hydraulic Data Collected Within the Period of Study

T_R(k)	T_a(k)	U_*(m/s)	$Vel_{(x)}$(m/s)	H (m)	B (m)	D_L (m^2/s)	D_T (m^2/s)
17.5	22.1	0.805	0.3439	1.654	10	6.364	0.7988
18.4	23.6	0.745	0.2793	1.787	10	4.863	0.7987
23.8	35.3	0.673	0.4537	1.556	10	9.298	0.628
24.5	30.1	0.745	0.3400	1.700	10	5.839	0.7599
24.6	30.4	0.681	0.3737	1.644	10	5.572	0.6717
25.1	30.2	0.745	0.4167	1.73	10	8.343	0.7731
25.6	29.3	0.682	0.2991	1.653	10	5.581	0.6764
26.6	35.4	0.687	0.3919	1.6431	10	7.417	0.6772

III. MODEL CALIBRATION AND VALIDATION

The model was calibrated using data obtained from the river under study 50% of the data was used for calibration of the data and 50% have been used for validating the data.

Table 1.0: Summarized Data for Model Calibration

Ta(k)	Ts(k)	u*(m/s)	Vel(m/s)	H(m)	B(m)	Experimental D_L(m^2/s)	Predicted D_L(m^2/s)
17.5	22.1	0.805	0.3439	1.654	10	6.902	6.9009
18.4	23.6	0.745	0.2793	1.787	10	5.488	5.4889
23.8	35.3	0.673	0.4537	1.556	10	9.120	9.1195
24.5	30.1	0.745	0.34	1.7	10	7.099	7.0998
						SQER =	0.0000
						RMSE =	0.0008
						R^2 =	0.9500

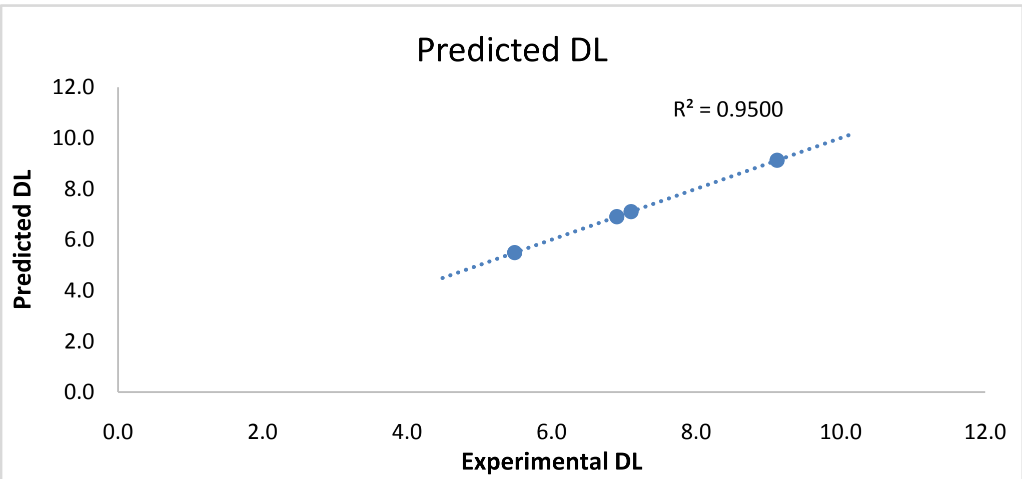

fig. 2: Predicted Dispersion Against Experimental Dispersion Values (model calibration)

Table 2.0: Summarized Data for Model Validation

a(k)	Ts(k)	u*(m/s)	Vel(m/s)	H(m)	B(m)	Experimental $D_L(m^2/s)$	Predicted $D_L(m^2/s)$
Ta	**Ts**	**u***	**vel**	**H**	**B**		
24.6	30.4	0.681	0.3737	1.644	10	7.358	8.1605
25.1	30.2	0.745	0.4167	1.73	10	8.713	9.0302
25.5	29.3	0.682	0.2991	1.653	10	6.020	6.6506
26.6	35.4	0.687	0.3919	1.6431	10	7.768	8.1764
						SQER =	1.3092
						RMSE =	0.5721
						R^2 =	0.9716

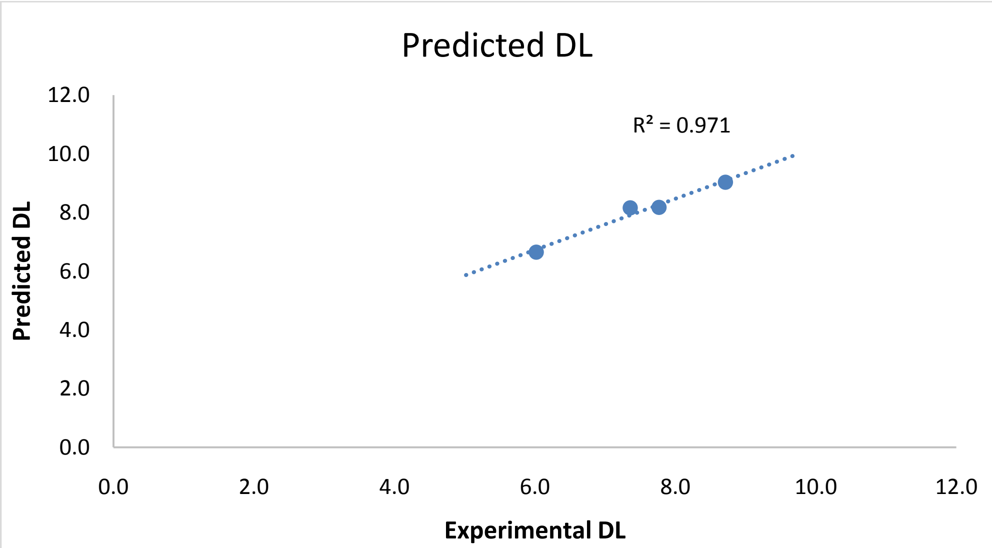

Fig. 3: Predicted Dispersion Against Experimental Dispersion Values (Model Validation)

Using appropriate regression analysis the model was calibrated thus:

$$D_L = \frac{3.1938B^{0.9229}V^{1.1275}}{U_x^{0.3516}H^{0.01470}}\left(\frac{T_a}{T_R}\right)^{0.6567} \tag{13}$$

The model was evaluated for its performance and compared with existing conventional models in literature. The performance analysis include the determination of coefficient of correlation, Rootmean square error and their discrepancy ratios. These indicators have the following expression (Loague and Green 1991) Kashefipour and Falconer, 2002).

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(D_{predicted} - D_{measured}\right)^2}{n}} \tag{14}$$

$$DR = \log_{10} \frac{D_p}{D_m} = \log_{10} D_p - \log_{10} D_m \tag{15}$$

$$CC = \frac{\sum\left(\overline{D}_{measured} - D_{measured}\right)^2 - \sum\left(D_{measured} - D_{predicted}\right)}{\sum\left(\overline{D}_{measured} - D_{measured}\right)^2} \tag{16}$$

Where $D_{predicted}$ and $D_{measured}$ are calculated and calculated and predicted dispersion coefficients. $\bar{D}_{measured}$ is the average dispersion coefficient measured.

IV. DISCUSSION

The model predicts fairly well the dispersion coefficient in natural stream. The model based on dimesnsional analysis incorporate temperature as a key factor that can affect dispersion. It is a common knowledge from kinetic theory that molecular diffusion increases with increase in temperature, hence the justification of temperature as a factor. Based on the discrepancy ratio, when the DR = 0 then it is an exact prediction, otherwise it is either an overestimation (DR >0 or Dp > Dm) or under estimation (DR < 0 or Dp < Dm). Negative values shows that the predicted values are much smaller than the measured value, while the positive values indicate that the predicted values are greater. The new model was compared with some of the other existing model in literature. The model was compared based on their coefficient of correlation, root mean square error and discrepancy ratio. In the comparison, results shows that the proposed model and Seo-Cheong (1998) over estimates the value of the dispersion coefficient since DR value are greater than zero. (see table below) and all other model underestimate the dispersion coefficient since their DR value is less than zero. The proposed model gave a reliable coefficient of correlation value and relatively low DR value. It indicates that the models are reliable for prediction. RMSE value of the proposed model was closest to zero. The lower the percentage error the better the model. The new model gave a percentage error of 10.9% which is not too bad. In accessing the performance of the model based on the error analysis, it may be inferred that the proposed model and model by Seo and Cheong (1998) perform in almost similar pattern.

Table 3: Formulation And Model Performance Based On R^2, RMSE, And DR

S/No	Model	Formulation	R^2	RMSE	DR
1	Liu 1977	$0.18\left[\frac{W}{H}\right]^2\left[\frac{u}{u_x}\right]^{0.5} HU_x$	0.8901	2.2332	-0.292
2	Fisher's (1979)	$= \frac{0.01Iu^2w^2}{hU_x}$	0.8263	7.3948	-1.726
3	Seo and Cheong (1998)	$D_l = 5.915\left[\frac{W}{h}\right]^{0.620}\left[\frac{u}{u_x}\right]^{1.428} hU_x$	0.9574	1.1127	0.0639
4	Kashefipour & Falconer (2002)	$D_l = \left[7.428 + 1.775\left[\frac{W}{h}\right]^{0.620}\left[\frac{u_x}{u}\right]^{0.572}\right] hU_x \left[\frac{u}{u_x}\right]$	0.9581	2.4678	-0.159
5	Sahay & Dutta (2009)	$2\left[\frac{W}{H}\right]^{0.96}\left[\frac{u}{u_x}\right]^{1.25} HU_x$	0.9488	1.5915	-0.088
6	Proposed model	$D_l = \frac{3.1938B^{0.9229}V^{1.1275}}{u_x^{0.3516}H^{0.01470}}$	0.9716	0.5721	0.0454

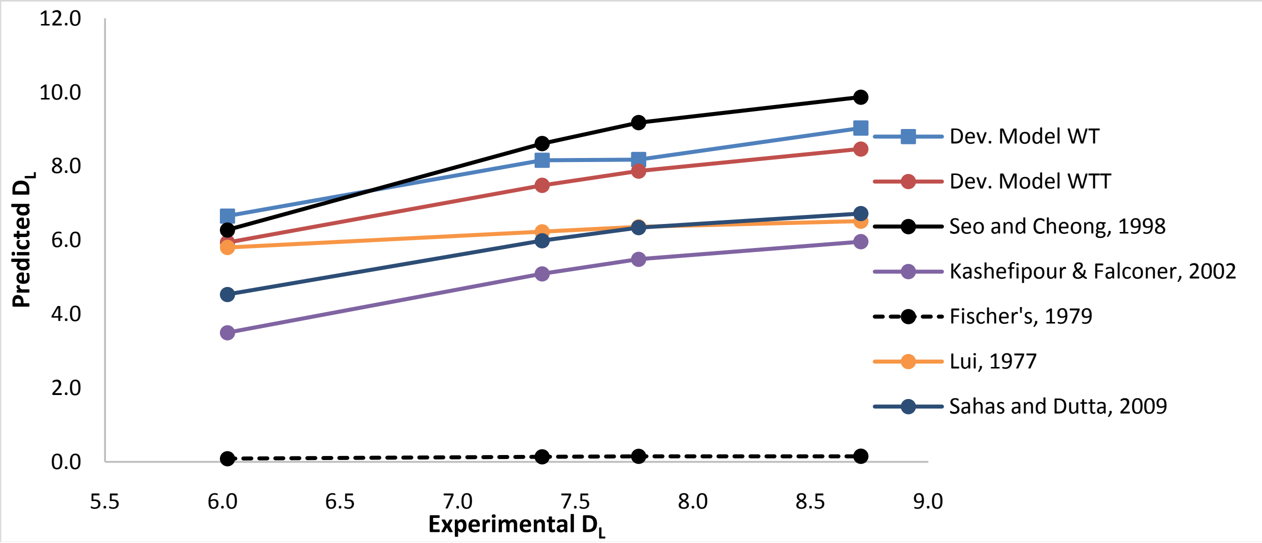

Fig. 4: Predicted Against Experimental Dispersion Coefficient for Different models

V. CONCLUSION

Model has been developed to estimate dispersion coefficient in natural stream incorporating temperature into the model. Water bodies undergo temperature variation along with normal climatic variations which occur seasonally. The temperature of the surface water is influenced by latitude, altitude, time of the day, time of the year, the flow and depth of the water body. Temperature affects also the physical, chemical and biological process in rivers and therefore values of many variables.

The performance has been evaluated and the model predicts fairly well the dispersion in streams. The correct determination of this variable are very key factors in developing a functional model. The model is strongly recommended for use in stream pollution modelling.

Finally, stream pollution modelling is now a subject of growing public concern and development of appropriate model for stream and river water management would be a step in the right direction.

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