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Inherent Characteristic Analysis of Continuous Beam under Axial Force

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Abstract

In order to solve the vibration of continuous beam which in the practical engineering application and reaches the influencing factors and change regular pattern, analyzed the natural frequency and vibration mode in the free modal, axial tension, axial compression based on Rayleigh theory. The specific change law is obtained by changing the size of the axial force and verified the rationality of the method though ANSYS. Finally, it is concluded that without considering the nonlinear condition, there is a liner relationship between inherent frequency and axial force, and its inherent frequency will increase gradually along with the increase of axial tension, and decrease gradually along with the increase of axial tension. The conclusion lays the theoretical foundation for the study of the inherent characteristics of the beam and the design of its engineering application.

Keywords: *continuous beam，Rayleigh theory，inherent frequency，vibration mode，axial force*

I. INTRODUCTION

With the rapid development of high technology, rotating machinery is widely used in various economic fields.While beam as the most basic component of a mechanical structure,has wider application background. Due to the influence of the extraneous factors, beam will get axial force on-stream and that will producethe influence of the vibration characteristics[1].

For the vibration of the beam under the axial force, QiboMao[2]discussed the free vibration and stability of double beam though Adomian correct decomposition method. LanmanGuoetc[3]、Boiangiuetc[4]analyzed the vibration and stability of intelligent beam with piezoelectric patches through the transfer matrix method. YinghuiLi[5] presented a fast calculation method for transverse vibration of the beam with variable axial forces. MeridaMohammadnejad[6] presented an analytic procedure of natural frequency when consider the axial bending and reversing. ShanliangLiu[7] analyzed the theoretical model of bearing beam with different boundary conditions and discussed it's parametric resonance and internal resonance. Above research, they didn't provide the concrete variation law of the beam's inherent characteristics. Therefore, based on the Rayleigh theory, the paper analyzes the vibration characteristics of a simple beam under tension and pressure. At last, the variation law of the natural frequency is achieved by changing the axial force.

II. MODAL ANALYSIS OF CONTINUOUS BEAMUNDER FREE VORTEX

The paper used the continuous beam model with simply supported.The specific simplified model and the physical parameters of thematerials are shown in Figure 1 and Table 1 respectively:

Fig.1A Simplified Model of Beams

Table 1 Physical Parameters of Materials

material	Aluminiumalloy
diameter	0.05m
length	1m
density	2710kg/m^3
Elasticmodulus	7×10^{4}MPa

Without considering the geometric nonlinear factors, from the constitutive relation and displacement compatibility condition,

$$M = EI\frac{\partial\theta}{\partial z}, \theta = \frac{\partial w}{\partial z} \quad (1)$$

Which gets the dynamic equation[8] of a uniform beam under free vortex,

$$-EI\frac{\partial^4 w}{\partial z^4} + \rho I\frac{\partial^4 w}{\partial z^2\partial t^2} - 2\Omega\rho Ii\frac{\partial^3 w}{\partial z^2\partial t} - \rho A\frac{\partial^2 w}{\partial t^2} = 0$$

(2)

In which, EI is the equivalent stiffness of the beam, ρ is the density, $w(z,t)$ is the lateral drift, Ω is the eddy velocity.

Order the equations , $w(z,t) = Y(z)e^{i\omega t}$ （3）

$$\begin{cases} \dfrac{2r}{l^2} = \dfrac{\rho\omega^2 - 2\Omega\rho\omega}{E} \quad (4) \\ \dfrac{\lambda^4}{l^4} = \dfrac{\rho A}{EI}\omega^2 \end{cases}$$

Therefore , the general solution of the equation is

$$Y(z) = c_1\sin\alpha z + c_2\cos\alpha z + c_3\sinh\beta z + c_4\cosh\beta z \quad (5)$$

Considering the boundary condition

$$\begin{cases} w(0) = 0, w''(0) = 0 \quad (6) \\ w(l) = 0, w''(l) = 0 \end{cases}$$

After normalization, which gets the equation of the natural frequency under free vortex state is

$$\omega_n = \left(\frac{n\pi}{l}\right)^2\sqrt{\frac{EI}{\bar{m}}} \quad (7)$$

The vibration equation is

$$Y(z) = \sin\frac{n\pi}{l}z \quad (8)$$

Thus, the first three natural frequencies are shown in table 2:

Table 2 Inherent frequency of simply supported beam under free stateHz

Natural frequency	Numerical value
f_1	126.135
f_2	504.54
f_3	1135.214

III. MODAL ANALYSIS OF THE CONTIONUOUS BEAM UNDER AXIAL FORCE

The simplified model of a uniform beam under axial force is shown in figure 2:

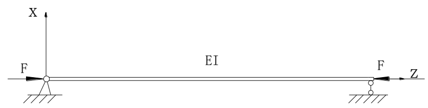


Fig.2Simplified model of the beam under axial force

In the paper, stipulating the axial pressure is positive and the tension is negative. The vorticity equation is $\frac{\partial^2}{\partial z^2}[EI(z)\frac{\partial^2 w}{\partial z^2}] + F_0\frac{\partial^2 w}{\partial z^2} + \rho A\frac{\partial^2 w}{\partial z^2} = 0$ (9)

Which EI is the equivalent stiffness of the beam, F_0 is the axial force, $w(z,t)$ is the lateral displacement.

Simplified form and decree $\lambda^2 = \frac{F_0}{EI}, \beta^4 = \frac{\rho A\omega^2}{EI}$ (10)

$$\begin{cases} \alpha_1 = \sqrt{\dfrac{\lambda^2}{2} + \sqrt{\dfrac{\lambda^4}{4} + \beta^4}} \quad (11) \\ \alpha_2 = \sqrt{-\dfrac{\lambda^2}{2} + \sqrt{\dfrac{\lambda^4}{4} + \beta^4}} \end{cases}$$

Simultaneous (6)、(9)、(10)、(11), gettingthe vibration equation is

$$w(z) = \sin(\frac{n\pi}{l}z)A\sin(\omega t + \theta) \quad (12)$$

The inherent frequency equation is

$$\omega = \omega_n\sqrt{1 - \frac{F}{n^2F_c}} \quad (13)$$

In which, n= (1, 2, 3…), $F_c = \frac{EI\pi^2}{l^2}$ is the critical load of the beam [9]. Through (13), we can know that the first natural frequency is close to zero when the axial force reaches the critical load. So it will engender pressure stability. Through (13), we can receive the first three natural frequency of the beam under axial tension and compression, as shown in Table 3 and Table 4.

Table3 Inherent frequency of continuous beam under axial pressure

Hz

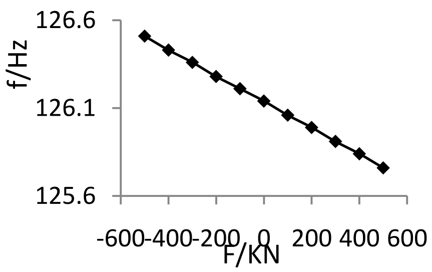
F \ f	f_1	f_2	f_3
100KN	126.06	504.47	1135.14
200KN	125.99	504.39	1135.06
300KN	125.91	504.32	1134.99
400KN	125.84	504.24	1134.91
500KN	125.76	504.17	1134.84

Table4:Inherent frequency of continuous beam under axial tension

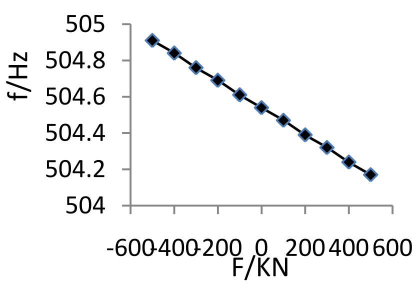
Hz

F \ f	f_1	f_2	f_3
100KN	126.21	504.61	1135.29
200KN	126.28	504.69	1135.36
300KN	126.36	504.76	1135.44
400KN	126.43	504.84	1135.51
500KN	126.51	504.91	1135.59

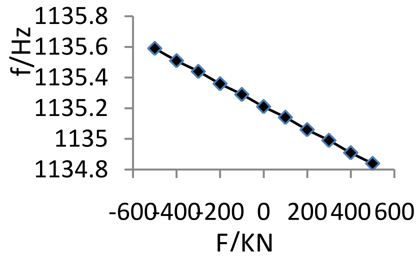
Above date, we can clearly chalk up the change regulation of the inherent frequency of a beam under different axial forces.As shown in Figure 3:



(a) First-order inherent frequency



(b) Second-order inherent frequency

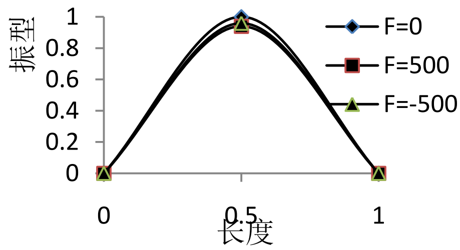


(c)Third-order inherent frequency

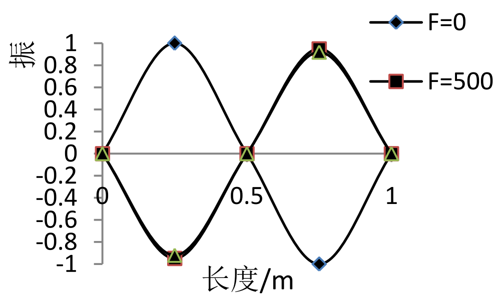
Fig.3 Variation law of inherent frequency

Above the drawn, we can know the natural frequency is approximately linear with the change of axial force when without discarding the nonlinear factors. Concretely, the natural frequency increases with the increase of axial tension and decreases with the increase of axial pressure. Thus, the axial pressure will reduce the natural frequency and the axial tension will improve the frequency. Finally, it can be seen that the change of the axial force have a great influence on the first-order natural frequency.

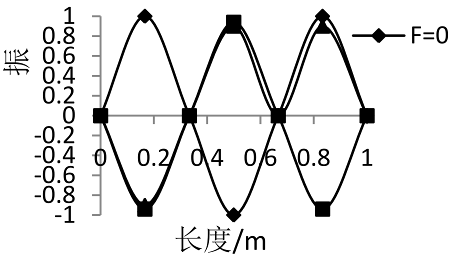
When the natural frequency changes, the vibration mode will change accordingly. The following compared the vibration modeunder free modal、tension is 500KN and pressure is 500KN. As shown in Figure 4:



(a)First-order mode



(b)Second-order mode



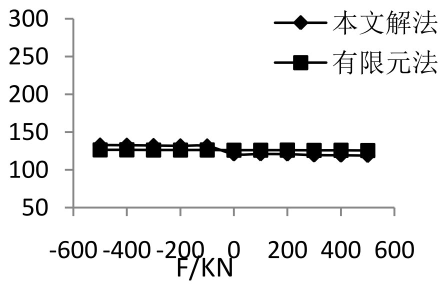
(c) third-order mode

Fig.4 vibration pattern

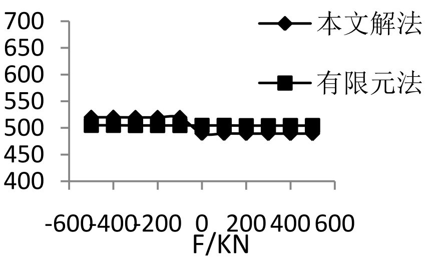
From the vibration pattern, it can know that the amplitude will decrease when the axial force is applied to the continuous beam. Compared pressure, the tension has little influence on the vibration mode. In addition, axial force has little influence on the first-order vibration mode and has a greater influence on the second and the third vibration mode.

IV. FINITE ELEMENT ANALYSIS

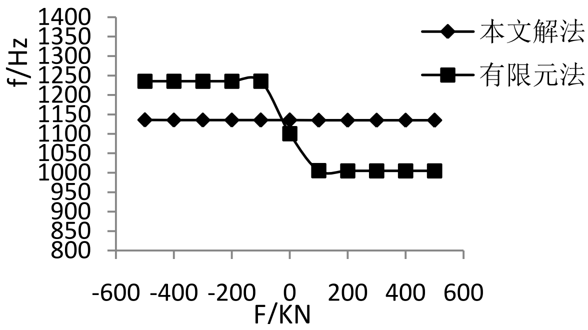
The basic idea of the finite element method is the continuous system with an infinite degree of freedom is approximated a discrete system with finite degrees of freedom [10]. Firstly, SolidWorks, a 3D software, is applied to establish the geometric model. Secondly, Workbench to mesh the model and applied load on its[11]. Finally, comparing the simulation value and the theoretical value of the natural frequency, the change rule is shown in Figure 5:



(a)First-order inherent frequency



(b)Second-order inherent frequency



(C)Third-order inherent frequency

Fig.5 Inherent Frequencies Under Different Algorithms

Analysis of figure, the natural frequency of the two algorithms has the same variation tendency under the axial force. With the increase of the axial force, the error will increase. From the figure, we can know the two algorithms have little influence on the first-order and the second-order natural frequency and their error is about 6%. But they have much influence on the third order natural frequency and its error is about 9%. Therefore, the rationality of the method can be verified.

V. CONCLUSION

(1) Based on Rayleigh theory, it solved the natural frequency and the mode under free modal, axial tension and axial pressure. Through finite element method verified the calculation error is within 10%.

(2) The changing law of the natural frequency is obtained when the axial force is gradually changing from tension to pressure and it is approximately linear when discarding the nonlinear factors. Analyzed the vibration mode of the beam under the free modal, the axial tension is 500KN and the pressure is 500KN and reached the tension has a smaller impact on the first mode and has a larger impact on the high mode.

(3) When the axial force reaches the critical load, the first-order natural frequency tends to zero. So it will engender the Stabilization of Compressive Bar.

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