SSRG International Journal of Recent Engineering Science
ISSN: 2349–7157 / https://doi.org/10.14445/23497157/IJRES-V11I4P101

Volume 11 Issue 4, 1-8, Jul-Aug 2024

Original Article

Non-Linear Regression Curve Fitting of Time-Dependent Growth Performance of Cobb500 Broiler

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Received: 11 May 2024 Revised: 22 June 2024 Accepted: 12 July 2024 Published: 02 August 2024

Abstract *- The aim of this study was to evaluate the performance of three different mathematical models (Gompertz, Von Bertalanffy, and Logistic) in predicting the growth of broiler chickens. The coefficients for each model were estimated using the MS solver, and the results were compared to values reported in previous studies. The body weight of the broilers was found to not show parallelism based on sex profile, with differences in weight observed starting from the 21st day of breeding. The Gompertz model was found to give the best prediction of the average body weight of male and female broilers, with a high correlation coefficient and model efficiency. These findings suggest that the Gompertz model is a good fit for predicting the growth of broiler chickens and that sex-specific growth patterns should be considered in future studies.*

Keywords *- Mathematical models, Regression, Broiler, Body weight, Prediction.*

1. Introduction

Non-linear regression curve fitting is a statistical technique that is commonly used to model time-dependent growth patterns. This method involves fitting a non-linear equation to a set of data points in order to capture the underlying trends and patterns in the data. Non-linear regression curve fitting has been widely applied in a variety of fields, including biology, economics, and engineering. In the field of biology, for instance, non-linear regression has been used to model the growth of bacterial cultures (Widder & Brückner, 2013), the development of fruit (Lu & Arora, 2006), and the growth of plants (Egea-Cortines & Zobel, 2013). One of the advantages of non-linear regression is that it can capture complex and non-linear patterns in the data that may not be apparent when using other statistical methods. Furthermore, non-linear regression allows researchers to evaluate the impact of different factors on the growth process, such as temperature, nutrient availability, and environmental conditions. Overall, non-linear regression curve fitting is a powerful tool for modeling time-dependent growth patterns and has been widely applied in a variety of fields. The Gompertz, logistic, and Bertalanffy growth models are mathematical equations that are commonly used to predict the body weight of broiler chickens. These models are based on different assumptions about the underlying growth process, and each has its own strengths and limitations. The Gompertz model is based on the assumption that growth follows a sigmoid curve, with a slow initial growth rate that accelerates as the chickens mature (Sánchez & Páramo, 2008). This model has been widely applied in studies of broiler growth and has been shown to provide accurate predictions of body weight over time (Hashemi & Ahmadi, 2010) The logistic model, on the other hand, is based on the assumption that growth follows a logistic curve, with an initial rapid growth rate that slows down as the chickens reach their maximum body weight (Lomax, 2006). This model has also been applied in studies of broiler growth and has been found to provide accurate predictions of body weight (Ferreira & Gomide, 2008). The Bertalanffy model is based on the assumption that growth follows an exponential curve, with a constant growth rate over time (Bertalanffy, 1938). This model has been used in a variety of applications, including the prediction of broiler body weight (Nkrumah & McNeill, 2002). Overall, the Gompertz, logistic, and Bertalanffy growth models are valuable tools for predicting the body weight of broiler chickens.

Each model has its own strengths and limitations, and the appropriate model to use in a given situation will depend on the specific growth characteristics of the chickens being studied. Animal development is a complex physiological and morphological process from hatching through maturity. Kaplan and Gürcan (2018) describe it as the increase in body weight and organ size per unit of time or age. Biologists who defined weight-age relationships in growing animals used various development models.

These models were used to predict mineral feeding requirements, optimal slaughter, and maturity age of broilers (Darmani-Kuhi et al., 2010; Kaplan & Gürcan, 2018). Mata-Estrada et al. (2020) compared the performance of Gompertz-Laird, logistic, Richards, and Von Bertalanffy's growth models on the body weight of Creole and local chicken. They discovered that male Creole chickens had a greater asymptotic weight than females. Segura-Correa et al. (2004) and Okeno et al. (2012) discovered that the body weight of Creole hens is nearly three times that of commercial breeds.

Many studies investigated the growth of local chickens using Gompertz, logistic, Richards, and Von Bertalanffy models (Yang et al., 2006; Osei-Amponsah et al., 2014; Zhao et al., 2015; Mata-Estrada et al., 2020). However, each nonlinear model has its own set of characteristics and limitations. Divergence and misunderstanding of model parameters are typical challenges caused by multiple reparameterizations of growth models reviewed by various studies (Tjørve and Tjørve, 2017). As a result, they become difficult to solve. Therefore, this study aimed to improve some model parameters and compare the Gompertz, Logistic, and Von Bertalanffy growth models to see which one best fits the Cobb500 broiler development pattern.

2. Materials and Methods

The study was simulated using the published data on Cobb500 broilers (Cobb, 2018). In the study, the body weight of broilers was studied for sixty-three days to determine their performance based on feed intake. The birds were fed 180 g per bird starter feed between 0 – 8 days, 700 g per bird grower feed between 9 – 18 days, 1350 g per bird finisher-1 between 19 – 28 days, and finisher-2 were fed to the birds between 29 – 63 days during the growout period (Cobb, 2018; Akinbobola, 2018). To determine the best-fitted growth model to predict the birds' body weight, three non-linear regression models were selected, namely Gompertz, Von Bertalanffy, and the logistic model.

2.1. Model Parameters Evaluation

The models were modified to have similar coefficients to evaluate the body weight of the broilers. Equations 1 to 3 show the state variable and parameters in the models (Kucukonder et al., 2020)

2.1.1. Gompertz Model

$$BW_t = \beta_0 \exp\text{-}\beta_1 \exp(-\beta_2 t) \quad (1)$$

2.1.2. Bertalanffy Model

$$BW_t = \beta_0 (1- \beta_1 \exp(- \beta_2 t))^3 \quad (2)$$

2.1.3. Logistic Model

$$BW_t = \beta_0 (1+ \beta_1 \exp\text{-} \beta_2 t)^{-1} \quad (3)$$

Where BW_t is the broiler's body weight (g) at age t (day). β_0 is the asymptotic weight (g), β_1 is the scaling parameter, and β_2 is the daily growth rate. Since it has been reported that the parameters of these models are challenging to solve (Tjørve & Tjørve, 2017), Microsoft Excel Solver was used to estimate the optimal parameter values. The objective was to minimize the Mean Absolute Error (MAE) while β_0 was defined, but β_1 and β_2 were solved to obtain the best value.

2.2. Statistical Analysis

The selected models were evaluated using some statistical metrics given below.

$$\text{Mean Absolute Error (MAE)} = \frac{\sum |Xi - Yi|}{N} \quad (4)$$

Mean Absolute Percentage Error

$$(\text{MA\%E}) = \frac{1}{N} * \sum_{i=1}^{N} \left| \frac{Xi - Yi}{Xi} \right| \quad (5)$$

Root Mean Square Error

$$(\text{RMSE}) = \frac{\sqrt{\sum_{i=1}^{N} (Xi - \bar{X}i)^2}}{N} \quad (6)$$

Normalized Root Mean Square Error

$$(\text{NRMSE}) = \frac{RMSE}{Xmin - Xmax} \quad (7)$$

Correlation coefficient

$$(\text{R}) = \frac{n \sum XY - (\sum X)(\sum Y)}{\sqrt{n(\sum X^2) - (\sum X)^2} * \sqrt{n(\sum Y^2) - (\sum Y)^2}} \quad (8)$$

Coefficient of Determination (R^2) (9)

Model efficiency

$$(\text{ME}) = 1 - \frac{\sum_{i=1}^{N} (Xi - Yi)^2}{\sum_{i=1}^{N} (Xi - \underline{X}i)^2} \quad (10)$$

Index of agreement

$$(\text{IA}) = 1 - \frac{\sum_{i=1}^{N} (Xi - Yi)^2}{\sum_{i=1}^{N} (|Yi - \underline{X}i| + |Xi - \underline{X}i|)^2} \quad (11)$$

where, N = number of data pairs, Xi = ith observed value, Yi= ith simulated value, X = value of measured data point, Y = value of simulated data point, $\bar{\bar{X}}$ = average of measured values.

3. Results and Discussion

The coefficients β_0 were given as the peak body weight during the study period, β_1 and β_2 were evaluated using MS solver and the values were given in Table 1. In terms of β_1 (scaling parameter), the Logistic regression model gave the highest estimated value in males and females compared to other models. Narinç et al. (2014) and Olamide Durodola, 2021 reported that the Gompertz model β_1 parameter values for slow-growing female and male broiler chicks are 4.42 and 4.74, respectively. In terms of β_2 (daily growth rate), the Logistic model gave the highest instantaneous growth parameter value. Gompertz's model predicted 0.053 and 0.054 for males and females, while Von Bertalanffy predicted 0.039 for both male and female broilers, respectively. These outputs were the lowest among the three models, and it was similar to the findings of Kucukonder et al. (2020).Furthermore, Using Von Bertalanffy's function, Yakupolu and Atil (2001) evaluated the development of Cobb and Hubbard commercial broilers. They found that the β_0, β_1, and β_2 parameters for females are 4923 g, 0.97, and 0.16, and for males are 5156 g, 0.99, and 0.17. Darmani-Kuhi et al. (2003) observed that the β_0 parameter for the Von Bertalanffy is 5159 g for females and 5475 g for male broilers, whereas for Logistic models is 3739 g for females and 4413 g for males. Parameter estimates from the Von Bertalanffy and Logistic models in this research are comparable to those from Yakupolu and Atil (2001) and Darmani-Kuhi et al. (2003).Figure 1 shows that the body weight of the broilers did not show parallelism based on sex profile. Differences in body weight were observed starting from the 21st day of breeding the broilers, and the dispersion continues to increase during growout. The findings were similar to Kucukonder et al. (2020), Marcato et al. (2008), and Narinç et al. (2007) observations using another commercial broiler breed. The growth data of the broilers were analyzed by Gompertz, Bertalanffy, and Logistic models shown in Figure 2.

Table 1. The estimated values of the growth model parameters

		Models		
		Gompertz	**Bertalanffy**	**Logistic**
	Model expression	β_0 exp-β_1 exp (-β_2 t)	β_0 (1- β_1 exp (- β_2 t))^3	β_0 (1+ β_1 exp- β_2 t)$^{-1}$
Male	β_0	5148	5148	5148
	β_1	4.643	0.800	91.063
	β_2	0.053	0.039	0.121
Female	β_0	4370	4370	4370
	β_1	4.473	0.787	76.991
	β_2	0.054	0.039	0.119

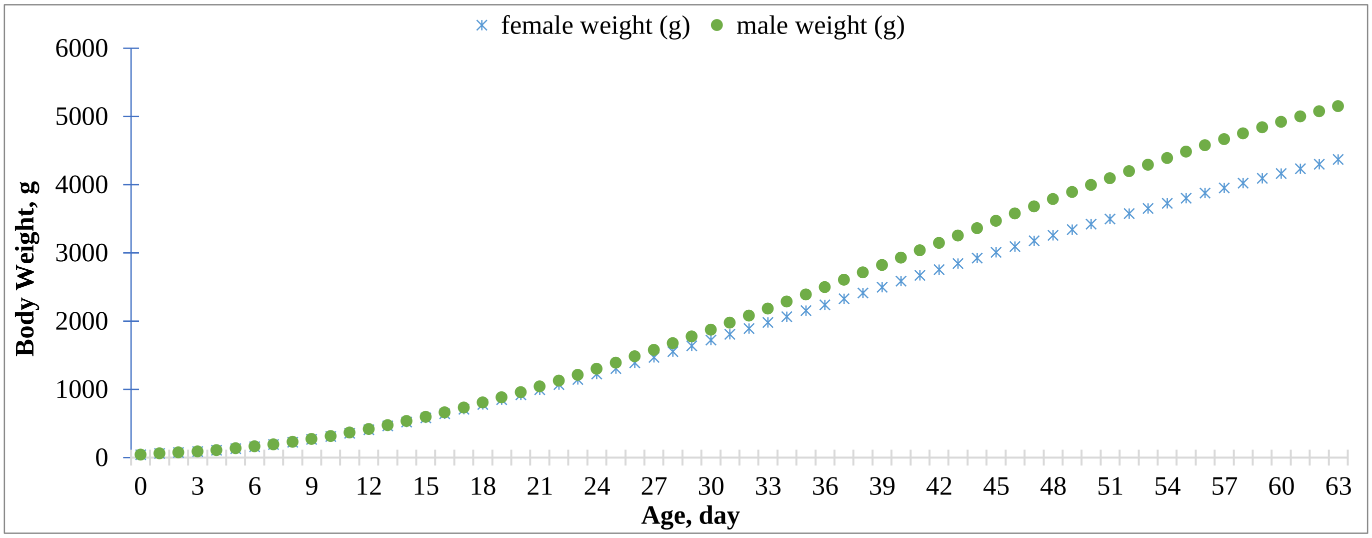
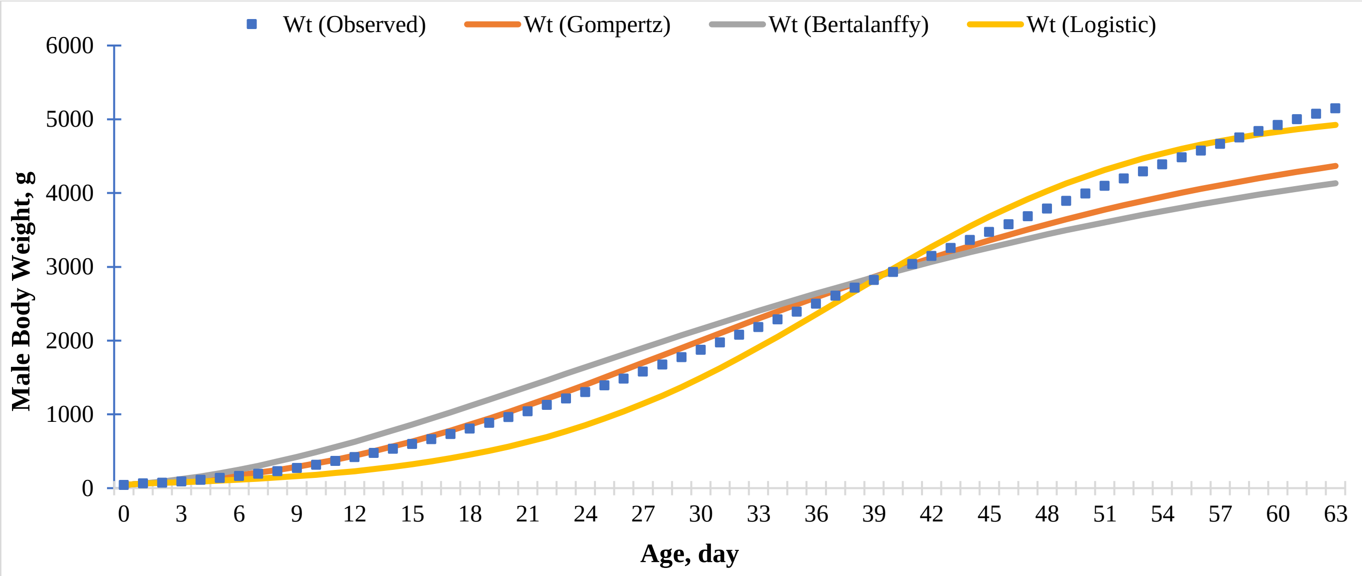
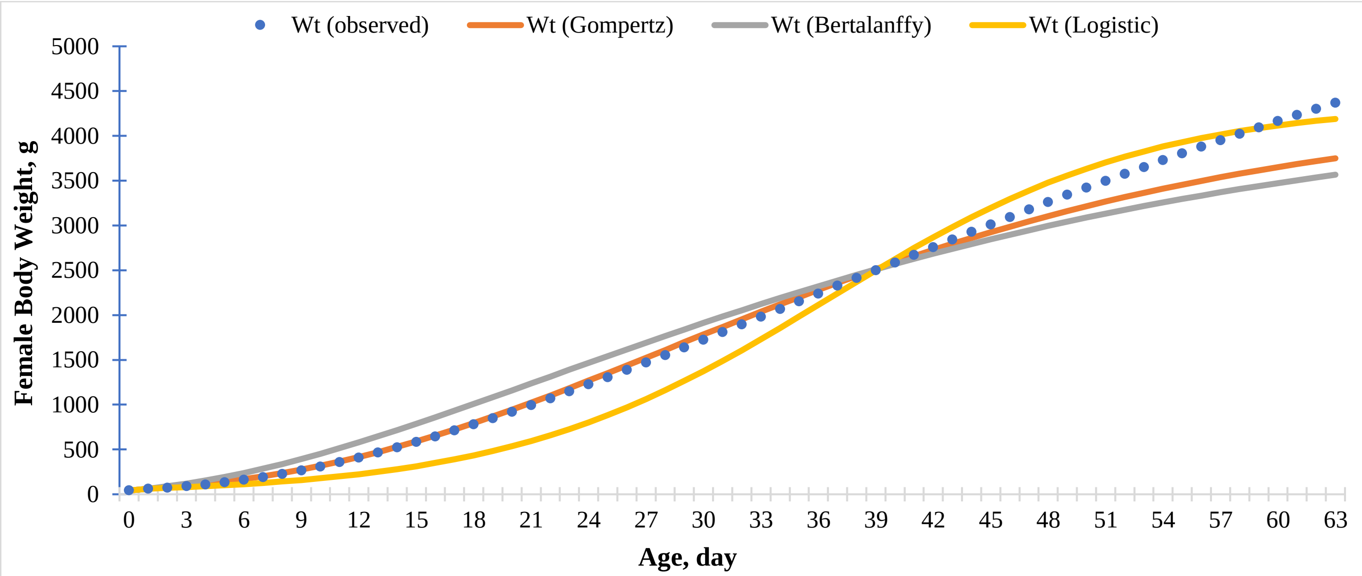


Fig. 1 Body weight profiles of female and male broilers

In the growth curve fitting, the Gompertz model gave the best fit at the early ages (1-40 days). Von Bertalanffy overfitted the data between this age, while the Logistic model underfitted the data. However, the three models gave accurate predictions at age 40 days for both male and female broiler body weight. The Gompertz and Von Bertalanffy models under-fitted the data at post-puberty (41-63 days). The pattern that emerged from the curve fitting gave the impression that the three models would function more effectively in a dual-phase mode, in which the model parameters would be solved based on the phases. Phase I would be represented by ages 1-40 days and Phase II by ages 41-63 days. Each function with two phases will be fitted to the data to predict the body weight of the broilers.



(a)



(b)

Fig. 2 Growth curve of broiler predicted by three models in comparison with the observed data (a) male, (b) female

Table 2. The performance of the three prediction models

		Observed	**Gompertz**	**Bertalanffy**	**Logistic**
Male	Avg. Weight (g)	2229.641	2134.497	2164.84	2120.523
	MAE (g)		172.20	320.01	201.12
	MA%E (%)		6.77	22.46	18.12
	RMSE (g)		273.00	411.83	244.41
	Correlation Coefficient (r)		0.99	0.99	0.99
	Coef. Of Determination (R^2)		0.99	0.97	0.99
	Model Efficiency		0.99	0.95	0.98
	Index of Agreement		0.99	0.98	0.99
	NRMSE (%)		12.24	18.47	10.96
Female	Avg. Weight (g)	1955.859	1871.54	1933.25	1855.45
	MAE (g)		117.18	239.40	188.78

	MA%E (%)		4.57	18.63	18.26
	RMSE (g)		203.25	311.04	230.28
	Correlation Coefficient (r)		1.00	0.99	0.99
	Coef. Of Determination (R^2)		0.99	0.98	0.99
	Model Efficiency		0.99	0.96	0.98
	Index of Agreement		0.99	0.98	0.99
	NRMSE (%)		10.39	15.90	11.77

According to Table 2, the Von Bertalanffy model gave the highest prediction of the average body weight of male and female broilers. The values predicted were closer to the average body weight observed. However, based on the metrics specified in Table 2, the Gompertz model gave the best prediction when evaluating the performance of the models. The correlation of coefficient, coefficient of determination, model efficiency, and index of agreement was 99% for all the performance metrics using equation 4 to equation 11.

4. Conclusion

The research looked at three different nonlinear models to analyze the pattern of body weight obtained by male and female broilers over 63 days. It was found that the Von Bertalanffy model provided an average body weight that was comparable to the actual data. However, when compared to the other growth functions, the Gompertz model provided the most accurate and satisfactory match. Since most researchers have claimed that it is challenging to assess the model parameter, this study concluded that the model parameters might be evaluated using Microsoft Excel Solver by reducing the mean absolute error as much as possible. Additionally, the pattern produced due to the curve fitting gave the impression that the three models would function more effectively in a dual-phase mode. This mode would be one in which the model parameters would be solved based on the phases to improve the curve fitting of the data. The ages 1-40 days would represent the first phase, while the ages 41-63 days would represent the second phase.

References

[1] A. Akinbobola, Broiler Feeding Guide, Growth & Weight Chart, 2018. [Online]. Available: https://www.livestocking.net/standard-broiler-feed-chart-growth-weight#Broiler_Starter_0-10_days

[2] L. Bertalanffy, "A Quantitative Theory of Organic Growth (Inquiries on Growth Laws. II)," *Human Biology*, vol. 10, no. 2, pp. 181-213, 1938. [Google Scholar] [Publisher Link]

[3] Cobb, Cobb500 Broiler Performance and Nutrition Supplements, 2022. [Online]. Available: https://www.cobbgenetics.com/assets/Cobb-Files/2022-Cobb500-Broiler-Performance-Nutrition-Supplement.pdf

[4] H. Darmani-Kuhi et al., "A Review of Mathematical Functions for the Analysis of Growth in Poultry," *World's Poultry Science Journal*, vol. 66, no. 2, pp. 227-240, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[5] H. Darmani-Kuhi et al., "An Evaluation of Different Growth Functions for Describing the Profile of Live Weight with Time (Age) in Meat and Egg Strains of Chicken," *Poultry Science*, vol. 82, no. 10, pp. 1536–1543, 2003. [CrossRef] [Google Scholar] [Publisher Link]

[6] M. Egea-Cortines, and R.W. Zobel, "Nonlinear Regression Analysis of Plant Growth Data," *Annals of Applied Biology*, vol. 162, no. 2, pp. 169-180, 2013.

[7] J.L.P. Ferreira, and J.A. Gomide, " Models for Broiler Chickens Growth Curve Fitting," *Revista Brasileira de Zootecnia*, vol. 37, no. 4, pp. 773-778, 2008.

[8] A. Hashemi, and M. Ahmadi, "A Comparison of the Gompertz and Logistic Models for Fitting Body Weight Data of Broiler Chickens," *World Applied Sciences Journal*, vol. 9, no. 7, pp. 827-831, 2010.

[9] Selcuk Kaplan, and Eser Kemal Gürcan, "Comparison of Growth Curves using Non-linear Regression Function in Japanese Quail," *Journal of Applied Animal Research*, vol. 46, no. 1, pp. 112-117, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[10] Hande Kucukonder et al., "Curve Fitting with Nonlinear Regression and Grey Prediction Model of Broiler Growth in Chickens," *Pakistan Journal of Zoology*, vol. 52, no. 1, pp. 347-354, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[11] M.A. Lomax, "The Use of the Logistic Function to Model Growth Data in Broiler Chickens," *Journal of Animal Science*, vol. 84, no. 7, pp. 1823-1830, 2006.

[12] Y. Lu, and A. Arora, "Nonlinear Regression Analysis of Apple Fruit Growth," *Scientia Horticulturae*, vol. 108, no. 3, pp. 295-303, 2006.

[13] S.M. Marcato et al., "Growth and Body Nutrient Deposition of Two Broiler Commercial Genetic Lines," *Brazilian Journal of Poultry Science*, vol. 10, no. 2, 2008. [CrossRef] [Google Scholar] [Publisher Link]

[14] Analy Mata-Estrada et al., "Comparison of Four Nonlinear Growth Models in Creole Chickens of Mexico," *Poultry Science*, vol. 99, no. 4, pp. 1995-2000, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[15] Dogan Narinç et al., "Analyses of Growth in Broilers from Different Growth Rate," *Journal of Animal Research*, vol. 17, pp. 1–8, 2007. [Google Scholar]

[16] Dogan Narinç et al., "Genetic Parameter Estimates of Growth Curve and Reproduction Traits in Japanese Quail," *Poultry Science*, vol. 93, no. 1, pp. 24-30, 2014. [CrossRef] [Google Scholar] [Publisher Link]
[17] J.D. Nkrumah, and D.M. McNeill, "A Review of the Application of the Von Bertalanffy Growth Model to the Growth of Broiler Chickens," *Journal of Applied Poultry Research*, vol. 11, no. 3, pp. 361-368, 2002.
[18] Tobias O. Okeno, Alexander K. Kahi, and Kurt J. Peters, "Characterization of Indigenous Chicken Production Systems in Kenya," *Tropical Animal Health and Production*, vol. 44, pp. 601-608, 2012. [CrossRef] [Google Scholar] [Publisher Link]
[19] Olamide Isaac Durodola, Comparison of Gompertz and Logistics Models of Broiler Growth, 2021. [CrossRef] [Publisher Link]
[20] R. Osei-Amponsah et al., "Evaluation of Models to Describe Temporal Growth in Local Chickens of Ghana," *Iranian Journal of Applied Animal Science*, vol. 4, no. 4, pp. 855-861, 2014. [Google Scholar]
[21] J.C. Sánchez, and F. Páramo, "Body Weight Prediction of Broilers using Nonlinear Models," *Journal of Applied Animal Research*, vol. 32, no. 1, pp. 25-33, 2008.
[22] J.C. Segura-Correa et al., "Productive Performance of Creole Chickens and Their Crosses Raised under Semi-intensive Management Conditions in Yucatan, Mexico," *British Poultry Science*, vol. 45, no. 3, pp. 342-345, 2004. [CrossRef] [Google Scholar] [Publisher Link]
[23] Kathleen M.C. Tjørve, and Even Tjørve, "The Use of Gompertz Models in Growth Analyses, and New Gompertz-model Approach: An Addition to the Unified-Richards Family," *PLOS ONE*, vol. 12, no. 6, p. e0178691, 2017. [CrossRef] [Google Scholar] [Publisher Link]
[24] S. Widder, and M. Brückner, "Nonlinear Regression Analysis of Growth Data of Bacterial Cultures," *Journal of Microbiological Methods*, vol. 95, no. 1, pp. 1-7, 2013.
[25] Cigdem Yakupoglu, and Hulya Atil, "Comparison of Growth Curve Models on Broilers Growth Curve I: Parameters Estimation," *Online Journal of Biological Science*, vol. 1, no. 7, pp. 680-681, 2001. [CrossRef] [Google Scholar] [Publisher Link]
[26] Y. Yang et al., "Analysis of Fitting Growth Models in Jinghai Mixed-Sex Yellow Chicken," *International Journal of Poultry Science*, vol. 5, no. 6, pp. 517–521, 2006. [Google Scholar] [Publisher Link]
[27] Zhenhua Zhao et al., "Comparative Study on Growth and Developmental Model of Indigenous Chicken Breeds in China," *Open Journal of Animal Science*, vol. 5, no. 2, pp. 219-223, 2015. [CrossRef] [Google Scholar] [Publisher Link]

Appendix

Table 1. Body weight prediction of Cobb500 Male broilers

Age (day)	Wt (Observed)	Wt (Gompertz)	Wt (Bertalanffy)	Wt (Logistic)
0	42	42	42	42
1	63	63	63	62.99995
2	74	79.08706	89.99085	70.96616
3	90	98.12268	122.354	79.92386
4	110	120.3915	160.0519	89.99219
5	135	146.1621	202.966	101.3035
6	164	175.6802	250.9143	114.0046
7	194	209.163	303.6648	128.2575
8	230	246.7938	360.9484	144.2413
9	271	288.7172	422.4683	162.1528
10	316	335.0363	487.9094	182.2074
11	365	385.8102	556.945	204.6405
12	418	441.0527	629.2429	229.7078
13	474	500.7324	704.4703	257.6857
14	534	564.7736	782.2979	288.871
15	597	633.0581	862.4031	323.5807
16	664	705.4279	944.4725	362.1501
17	733	781.6886	1028.204	404.931
18	806	861.6134	1113.308	452.288
19	882	944.9472	1199.51	504.5943
20	960	1031.411	1286.548	562.2253
21	1042	1120.707	1374.177	625.5518
22	1125	1212.524	1462.167	694.9303
23	1212	1306.538	1550.303	770.692
24	1300	1402.422	1638.385	853.131
25	1391	1499.848	1726.228	942.4895

26	1484	1598.486	1813.664	1038.944
27	1579	1698.017	1900.535	1142.587
28	1675	1798.127	1986.701	1253.415
29	1774	1898.514	2072.032	1371.314
30	1874	1998.889	2156.411	1496.042
31	1975	2098.978	2239.735	1627.227
32	2078	2198.524	2321.91	1764.357
33	2182	2297.287	2402.854	1906.785
34	2286	2395.045	2482.494	2053.732
35	2392	2491.594	2560.767	2204.303
36	2499	2586.75	2637.619	2357.501
37	2606	2680.347	2713.003	2512.26
38	2714	2772.237	2786.881	2667.467
39	2822	2862.289	2859.221	2821.997
40	2930	2950.392	2929.999	2974.742
41	3038	3036.448	2999.196	3124.649
42	3147	3120.378	3066.798	3270.742
43	3255	3202.115	3132.797	3412.146
44	3363	3281.607	3197.188	3548.108
45	3470	3358.817	3259.974	3678.009
46	3577	3433.716	3321.157	3801.366
47	3682	3506.29	3380.746	3917.834
48	3787	3576.532	3438.752	4027.199
49	3891	3644.446	3495.188	4129.368
50	3994	3710.043	3550.071	4224.36
51	4095	3773.343	3603.42	4312.287
52	4195	3834.371	3655.254	4393.338
53	4293	3893.158	3705.596	4467.769
54	4389	3949.742	3754.47	4535.882
55	4484	4004.163	3801.901	4598.014
56	4576	4056.467	3847.914	4654.525
57	4666	4106.699	3892.538	4705.789
58	4753	4154.912	3935.799	4752.18
59	4838	4201.158	3977.727	4794.07
60	4920	4245.491	4018.351	4831.822
61	4999	4287.966	4057.699	4865.784
62	5075	4328.64	4095.802	4896.288
63	5148	4367.569	4132.689	4923.647

Table 2. Body weight prediction of Cobb500 Female broilers

Age (day)	Wt (observed)	Wt (Gompertz)	Wt (Bertalanffy)	Wt (Logistic)
0	42	42	42	42
1	63	63.00004	63	63
2	74	78.59318	88.59212	70.81982
3	89	96.92118	118.9673	79.59233
4	108	118.2235	154.054	89.4289
5	133	142.7209	193.7131	100.4527
6	162	170.6103	237.753	112.7995
7	191	202.0594	285.9417	126.6189
8	227	237.203	338.018	142.0749
9	267	276.1394	393.6999	159.3467
10	310	318.9284	452.6921	178.6295
11	358	365.5901	514.6925	200.1348
12	409	416.1041	579.3969	224.0907

13	464	470.4111	646.5031	250.7417
14	521	528.4136	715.7145	280.348
15	582	589.9786	786.7426	313.1842
16	645	654.9403	859.3089	349.5377
17	711	723.1039	933.1467	389.7056
18	779	794.2489	1008.002	433.9913
19	849	868.1334	1083.635	482.6992
20	921	944.4983	1159.819	536.1292
21	995	1023.071	1236.342	594.5687
22	1071	1103.571	1313.008	658.2845
23	1148	1185.71	1389.632	727.5125
24	1227	1269.201	1466.047	802.447
25	1307	1353.757	1542.096	883.2282
26	1389	1439.096	1617.639	969.9306
27	1471	1524.945	1692.545	1062.551
28	1554	1611.037	1766.697	1160.995
29	1638	1697.121	1839.991	1265.071
30	1723	1782.957	1912.333	1374.481
31	1808	1868.319	1983.639	1488.817
32	1894	1952.997	2053.834	1607.559
33	1980	2036.798	2122.854	1730.084
34	2067	2119.545	2190.645	1855.676
35	2153	2201.076	2257.157	1983.534
36	2240	2281.247	2322.35	2112.798
37	2327	2359.93	2386.192	2242.571
38	2413	2437.012	2448.655	2371.939
39	2500	2512.396	2509.719	2500
40	2586	2585.999	2569.368	2625.888
41	2672	2657.752	2627.592	2748.797
42	2757	2727.599	2684.385	2867.999
43	2843	2795.496	2739.745	2982.859
44	2927	2861.41	2793.675	3092.847
45	3011	2925.32	2846.18	3197.542
46	3094	2987.214	2897.268	3296.633
47	3177	3047.087	2946.952	3389.917
48	3260	3104.944	2995.245	3477.288
49	3342	3160.797	3042.163	3558.735
50	3421	3214.663	3087.724	3634.323
51	3498	3266.567	3131.948	3704.186
52	3576	3316.536	3174.856	3768.513
53	3652	3364.604	3216.471	3827.535
54	3728	3410.808	3256.817	3881.515
55	3804	3455.186	3295.917	3930.741
56	3878	3497.783	3333.797	3975.511
57	3952	3538.643	3370.484	4016.128
58	4024	3577.811	3406.003	4052.896
59	4094	3615.336	3440.382	4086.115
60	4164	3651.267	3473.647	4116.071
61	4233	3685.654	3505.826	4143.043
62	4302	3718.545	3536.946	4167.29
63	4370	3749.991	3567.034	4189.06