Original Article

Winding of Distribution Transformer in Substations Models

Eyenubo O. J¹, Ebisne E. E², Okieke U. J³

^{1,2,3}Department of Electrical/Electronic, Delta State University, Abraka, Oleh Campus.

¹Corresponding Author : ojeyenubo@delsu.edu.ng

Received: 11 September 2023 Revised: 19 October 2023 Accepted: 07 November 2023 Published: 23 November 2023

Abstract - Distribution Transformers (DTs) in a substation are very essential paraphernalia. It builds reliable set-up and dependable. Transformers can breakdown in excess of period owed to various reasons such as additional load, insulation broken-down, roughage wear and tear, and weak dielectric strength. However, transformer merchants have enhanced the complexity in observing the function of DT significance which stops tradition breakdown. Surveys of overloaded DT mechanisms climax in the secondary interrelated Power Analyzer PCE-PA 8000. The technique for effecting new set-up of overloaded reliability capacities of DTs are 1000 kVA and 500 kVA, 11/0.415 kV used for investigation comprising administration and safety engineers' obligation subsequent to the analysed factors. The procedure of employing the acceded Power Quality (PQ) gadget offers a dependable brand-new method for evaluating the setup of DTs in a substation.

Keywords - Distribution Transformers, Power quality, Armature winding, Electrical magnitude, Harmonics.

1. Introduction

The windings in which voltage is induced are called armature windings. The winding in which the majority of the field flux is generated when the current through the winding is passed is referred to as the field winding.

Following are definitions for some of the fundamental words used in armature winding:

• Turn: A turn is a pair of conductors with an end connector joining them at one end.

- Multiple turns are strung together to form a helix.
- Winding: Several coils are connected in series to create a winding. The turn's figure is displayed below:



Fig. 1 The symbol (S) representing Start identifies the commencement the twist or helix, while the symbol (F) representing Finish identifies the end. The symbol (S) representing Start identifies the commencement of the twist or helix, while the symbol (F) representing Finish identifies the end.

The electrical magnitude is crucial to the study of machines.



Fig. 2 The symbol (S) represents start while (F) signifies finish of the turn or coil.

A (P) pole machine's electrical degree is described below

$$\theta_{ed} \triangleq \frac{P}{2} \theta_{md} \tag{1}$$

 θ_{md} is a way to measure angles in space or mechanical degrees.

 θ_{ed} is a measure of electrical angular velocity expressed in cycles or degrees.

The benefit the point of this note is that it may be used for machines with as many poles as you want using expressions specified in terms of electrical angles.

The angular separation between the centres of two successive poles of a machine is known as its pole pitch or pole span.



Fig. 3 The following diagram illustrates the short-pitch coil factor

Chorded coil is another name for it. The following diagram illustrates the short-pitch coil factor.



Fig. 4 A stator winding that utilizes fractional pitch-coil referred to as a chorded winding. If the coil span is diminished by an angle in electrical degrees, the coil's new span will be $(180^0 - \alpha)$ electrical degrees.

A stator winding that utilizes a fractional pitch-coil is referred to as a chorded winding. If the coil span is diminished by an angle in electrical degrees, the coil's new span will be $(180^{\circ} - \alpha)$ electrical degrees.

The separation between the two sides of a complete pitch-coil equates to 180⁰ electrical pole pitches. As a result, the voltages on each side of a full-pitch coil are in phase.

Let E_{C1} and E_{C2} denote the voltages produced in coil sides, and let E_C denote the voltage produced by the coil.

The resulting equation is shown below.

$$E_C = E_{C1} + E_{C2}$$

$$|E_{C1}| = |E_{C2}| = E_1 \ (Assumed)$$

The resulting coil voltages are in phase and equal to their arithmetic total.

So,

$$E_c = E_{c1} + E_{c2} = 2E_1$$

If a single coil's span is smaller than a transformer's pole pitch, 180° electrical, the voltages produced by each coil are out of phase. E_c , the resulting coil voltage equals the phasor sum of E_{c1} and E_{c2} .

Reducing the coil span by an angle of one electrical degree yields a new coil span of $(180^0 - \alpha)$ degrees. There will be a phase difference between the voltages produced by the two coil halves. In the above phasor diagram, the total of the phasors of and is represented by AC.

 $K_{C} = \frac{Actual \ voltage}{\frac{generated \ in \ the \ coil}{Voltage \ generated \ in \ the}}$ $coil \ of \ span \ 180^{\circ} \ electrical$

 $K_{C} = \frac{voltages of two coil sides}{Arithmentic sum of the}$ voltages of the two coil sides

$$K_{C} = \frac{AC}{2AB} = \frac{2AD}{2AB} = \cos\frac{\alpha}{2}$$
$$K_{C} = \cos\frac{\alpha}{2}$$

For a complete pitch-coil, the value of α is 0^0 ; $\cos \frac{\alpha}{2} = 1$ then, $K_c = 1$

1.1. Distribution Variable

Distribution factor (breadth factor) is the ratio of actual voltage to the maximum voltage that might have been achieved if all coils of a polar group were concerted in a definite slot. Its symbol and equation are below

$$K_{d} = \frac{\frac{coil \ voltages \ per \ phase}{Arithmentic \ sum \ of}}{\frac{coil \ voltages \ per \ phase}{coil \ voltages \ per \ phase}}$$
(3)

In a condensed zigzag, the coil's phases are all placed in one specific location. All of the induced coils' voltages are in sync with one another. Calculating these voltages calls for some mental math.

The induced voltage for a given phase is calculated by multiplying the voltage across a coil according to the strand count in that phase that is linked in series. By spreading out, they cluster together each pole's base.

The summation of phasors coil voltages equals the induced voltage in the coil side, which is out of phase by how far apart the slots are. Assume:

M = slots/pole/phase

$$m = \frac{slots}{poles \ x \ phases} \tag{4}$$

 β = angular displacement adjacent slots in electrical degrees

$$\beta = \frac{180^{\circ}}{\frac{slot}{poles}} = \frac{180^{\circ}x \, poles}{slot}$$
(5)

Thus, the phase of winding has successive coils $E_{C1}, E_{C2}, E_{C3}, \cdots$. Voltages represent coil.

Potential decreases. The slot pitch β determines the phase of each coil voltage.

The diagram below depicts the polygon of the induced voltages in a group of four coils. (m = 4)



Fig. 5 The voltages E_{C1} , E_{C2} , E_{C3} and E_{C4} are illustrated by phasors AB, BC, CD and DF individually. Every phasor subtends an angle β at O, a chord of the circle centred there. The phasor sum AF represents the resultant winding voltage delimits at an angle of 90 degrees.

The arithmetic sum of the voltages of the constituent coils is denoted as

$$mE_{c} = mAB = m(2AM)$$

$$= 2mOA \sin A OM = 2mOA \sin \frac{\beta}{2}$$

$$mE_{c} = mAB = m(2AM)$$

$$= 2mOA \sin A OM = 2mOA \sin \frac{\beta}{2}$$
Phasor sum of the coil
$$\frac{voltages \ per \ phase}{Arithmentic \ sum \ of \ coil} = \frac{2OA \sin \frac{m\beta}{2}}{2OAm \sin \frac{m\beta}{2}}$$

$$=\frac{\sin\frac{m\beta}{2}}{m\sin\frac{m\beta}{2}}$$
(6)

The number of distributed spaces associated with a pole determines a phase's distribution coefficient K_d . Lap or wave winding works the same. Slots per pole lower the dispersion factor.

1.2. Aspect of Winding

 K_d

In 3-phase AC machines, the winding factor improves the rms produced voltage to ensure that torque and output voltage are free of harmonics that would otherwise impair the machine's efficiency. Distribution factor (K_d) multiplied by the winding factor yields the winding factor. Coil span refers to the number of armature positions on either side of a coil, whereas the distribution factor is the proportion of the voltage produced by a distributed winding to a concentrated winding. The symbol for it is. The formula is given in the next section.

$$E_p = 4.44 K_w f \varphi T_p \tag{7}$$

The induced voltage is assumed to be sinusoidal; nonsinusoidal flux density distribution will result in nonsinusoidal induced voltage in the winding. Each harmonic's coil span, distribution, and winding factor will be unique. The following equation derives the fundamental *EMF* per phase from equation (7).

$$E_{p1} = 4.44 K_{w1} f \varphi_1 T_p \tag{8}$$

The *EMF* per phase for the 3^{rd} harmonic will be

$$E_{p3} = 4.44K_{w3}(3f)\varphi_3 T_p \tag{9}$$

The n^{th} harmonic of the *EMF* /phase will be

$$E_{pn} = 4.44 K_{wn}(nf)\varphi_n T_p \tag{10}$$

Here, the subscripts 1,3 and n correspond to the fundamental, third, and n^{th} harmonics, respectively.

Therefore,

$$\frac{E_{pn}}{E_{p1}} = \frac{K_{wn}}{K_{w1}} x \frac{n\varphi_n}{\varphi_1}$$
(11)

Where:

* φ_1 is the average flux multiplied by the area under one pole

$$\varphi_{1} = \left(\frac{peak flux density}{\frac{\pi}{2}}\right) x (area under one pole)$$
$$\varphi_{1} = \left(\frac{B_{m1}}{\frac{\pi}{2}}\right) \left(\frac{\pi DL}{P}\right)$$
$$\varphi_{1} = \frac{2DL}{P} B_{m1}$$
(12)

Where,

* B_{m1} is the maximum value of the basic flux density wave component.

* D is the armature's diameter or the average air gap width

* *L* is the length of the frame or side of the active coil

Likewise, for the nth harmonic

$$Pole \ pitch = \frac{\pi D}{P_n} \tag{13}$$

$$\varphi_n = \frac{2DL}{n^P} B_{mn} \tag{14}$$

Hence:

$$\frac{E_{Pn}}{E_{P1}} = \frac{K_{wn}B_{mn}}{K_{w1}B_{m1}} \tag{15}$$

Where:

 B_{mn} is the peak value of the n^{th} harmonic flux density.

1.3. Factor of Winding for the nth Harmonic

The expression for the winding factor equivalent to n^{th} harmonic voltage is:

$$K_{wn} = K_{cn} K_{dn} \tag{16}$$

 $K_{cn}K_{dn}$ are the coil and distribution factors, respectively, for the n^{th} harmonic

Consequently, the following equation describes the nthorder harmonic EMF induced per phase.

$$E_P = 4.44K_{cn}K_{dn}(nf)\varphi_n T_p \tag{17}$$

Where

$$\varphi_n = \frac{2DL}{nP} B_{mn} \tag{18}$$

A winding's induced voltage will comprise harmonics due to space flux density's non-sinusoidal distribution. Since flux density wave splits are identical, only odd harmonics are feasible. Phase voltage may contain 3^{rd} , 5^{th} , 7^{th} , and higher-order harmonics.

3-phase alternators are usually star-connected. Odd harmonic voltages match in amplitude and phase. The voltage across any two lines of a star-connected machine equals the phasor difference between the phases.'

Voltages. Thus, star-connected synchronous machines lack the 3^{rd} harmonic and its multiples in line voltage.

Since the intensity of voltage's harmonic components decreases with frequency, only the 5^{th} and 7^{th} harmonics are significant; this is referred to as Belt harmonics.

Since the intensity of voltage's harmonic components decreases with frequency, only the 5^{th} and 7^{th} harmonics are significant; this is referred to as Belt harmonics.

The following equation describes the r.m.s of induced-voltage across the lines of a 3-phase, star-connected machine.

$$E_{line} = \sqrt{3}x \sqrt{E_1^2 + E_5^2 + E_7^2 + E_{11}^2 + \cdots}$$

Where 1^{st} , 5^{th} , 7^{th} , 11^{th} , \cdots harmonics of the fundamental scale, respectively.

IEEE 519 serves as the recommended limit on harmonic distortion according to two distinct criteria, namely:

- There is a limitation on the amount of harmonic current that a consumer can inject into a utility network.
- A limitation is placed on the level of harmonic voltage that a utility can supply to a consumer.

2. Economic Consequences of Harmonics

The economic consequence of harmonic results in the increase of the r.m.s current value for different circuits and the deterioration of the supply voltage quality; the negative impact may remain unnoticed with its adverse economic results. Thus, that is why proper harmonic mitigation is needed to contribute to improving the competitiveness of companies in different ways:

- a. Reduced overloading of the electrical system.
- b. Reduced system loss and power demand.
- c. Reduced risk of outage.
- d. Extended equipment lifetime.
- e. Distortion Indices
- f. The most common measure of the power quality of a periodic waveform is the Total Harmonic Distortion (THD), which is expressed as the r.m.s value of the harmonics above fundamental, divided by the r.m.s fundamental worth. Thus, for current can be defined in the frequency domain in terms of the Fourier series coefficients.

$$THD_{1} = \frac{\sum_{k=2}^{\infty} \left(\frac{l_{k}}{\sqrt{2}}\right)^{2}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{\frac{1}{2}} \sum_{k=2}^{\infty} l_{k}^{2}}{\frac{1}{\sqrt{2}}}$$
(19)

The same equation form applies to voltage $THD_V THD_I$ and r.m.s is directly linked.

Note that since

$$I_{rms}^2 = \frac{1}{2} \sum_{k=2}^{\infty} I_k^2$$
 (20)

and since,

$$THD_1^2 = \frac{1/2\sum_{k=2}^{\infty} I_k}{\frac{l_1^2}{2}} = \frac{\sum_{k=2}^{\infty} I_k^2}{l_1^2}$$
(21)

then writing yields,

 $\frac{1}{2}\sum_{k=2}^{\infty}I_k^2 = I_1^2(1 + THD_1^2)$

so that,

$$I_{rms}^{2} = \frac{1}{2} \sum_{k=2}^{\infty} I_{k}^{2} = \frac{I_{1}^{2}}{2} (1 + THD_{1}^{2}) = I_{rms}^{2} (1 + THD_{1}^{2})$$

Thus,

the equation linking THD is,

$$I_{rms} = I_{1,rms} \sqrt{1 + THD_1^2}$$
 (22)

Effective value,

$$V_{RMS} = \sqrt{\frac{1}{T}} \int_0^T v(t)^2 dt - V_1 \sqrt{1 + THD_V^2}$$
(23)

$$I_{RMS} = \sqrt{\frac{1}{T}} \int_0^T i(t)^2 dt - I_1 \sqrt{1 + THD_i^2}$$
(24)

Harmonic currents can increase the rms current beyond what is needed to provide load power, and harmonic currents do not flow uniformly throughout the crosssectional area of a conductor, thereby increasing its equivalent resistance. Dry-type transformers are especially sensitive to harmonics; the K factor was developed to provide a convenient measure for rating the capability of transformers, especially dry types, to serve distorting loads without overheating.

$$K = \frac{\sum_{k=l}^{\infty} k^2 I^2_{k}}{\sum_{k=l}^{\infty} I^2_{k}}$$
(25)

in most situations, $K \le 10$

2.1. Harmonics Distortion and Fourier Series

Any periodic (repetitive) complex waveform is composed of a sinusoidal component at the fundamental frequency and a number of harmonic components, which are integral multiples of the fundamental frequency. The instantaneous value of voltage for a non-sinusoidal waveform or complex wave can be expressed as:

$$V = V_0 + V_1 \sin (\omega t + \varphi_1) + V_2 \sin (2\omega t + \varphi_2) + V_3 \sin (3\omega t + \varphi_3) + \cdots + V_n \sin (n\omega t + \varphi_n)$$
(26)

Where:

V = instantaneous value at any time t

 V_0 = direct (or mean) value (DC component)

 $V_1 =$ the essential component's rms value

 V_2 = the second harmonic component's rms value

 $V_3 =$ third harmonic component rms

 $V_n = r.m.s$ value of the n^{th} harmonic component

 $\varphi_n =$ relative angular frequency

$$\omega_0 = 2 \pi f_0$$

 f_0 = frequency of fundamental component (T/f defining the time over which the complex wave repeats itself).

According to Joseph Fourier-related techniques, any periodic function in a time interval could be expressed by the sum of the fundamental frequency and a number of higher-order harmonic frequencies, which are integral multiples of the fundamental component. Without accounting for any DC components, the instantaneous rms voltage can be expressed as a Fourier series using the above formula, where V_1 and I_1 stand for the fundamental voltage and current, respectively.

The frequency content of the time-domain signal is examined using the discrete Fourier transform technique (DFT). The Sampling Theorem states that for a proper information transfer to the sampled system, the sampling frequency of the data must be at least twice as high as the highest frequency present in the original signal. Nyquist frequency is the frequency at half sampling frequency. When the sampling frequency is less than twice the maximum frequency contained in the sampled waveform, the phenomenon known as "aliasing" might occur. Errors come from aliasing, which makes high-frequency components appear alongside real low-frequency ones.

The DFT of a *N*-sample sequence can be obtained as follows:

$$C_h = \frac{2}{N} \sum_{n=0}^{N-1} f[n] * e^{\frac{-j2\pi hn}{N}} = \frac{2}{N} \sum_{n=0}^{N-1} f[n] * W^{hn}$$
(27)

Where

h is the harmonic order, *n* is the data point number (*n* = 0 represents the 1st data point), *N* is the total sample points (in 1 cycle), and $W = e^{\frac{-j2\pi}{N}}$

$$\begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_{N-1} \end{bmatrix}_{N \times 1} = \frac{2}{N} \begin{bmatrix} W^{(0x0)} & W^{(0x1)} & \cdots & W^{(0x(N-1))} \\ W^{(1x0)} & W^{(1x1)} & \cdots & W^{(1x(N-1))} \\ \vdots & \vdots & \vdots \\ W^{((N-1)x0)} & W^{((N-1)x1)} & \cdots & W^{((N-1)x(N-1)} \end{bmatrix}_{N \times N} \begin{bmatrix} f[0] \\ f[1] \\ \vdots \\ f[N-1] \end{bmatrix}_{N \times 1}$$

$$\begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_{N-1} \end{bmatrix} = \frac{2}{N} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & W & \cdots & W^{N-1} \\ \vdots & \vdots & \vdots \\ 1 & W^{N-1} & \cdots & W^{((N-1)x(N-1)} \end{bmatrix} \begin{bmatrix} f[0] \\ f[1] \\ \vdots \\ f[N-1] \end{bmatrix} = \frac{2}{N} [D] * \begin{bmatrix} f[0] \\ f[1] \\ \vdots \\ f[N-1] \end{bmatrix}$$
(28)

Harmonic magnitudes and phase angles can be found from absolute and phase values of complex coefficients C_n .

The most commonly used measure of the quality of a periodic waveform is the total harmonic distortion (THD) THD is determined by dividing the rms value of the fundamental by the rms square value of the harmonics above it. As a result, current can be described using the Fourier series coefficients in the frequency domain.

$$v(t) = \sum_{h=2}^{\infty} V_h(t) = \sum_{h=2}^{\infty} 2V_h \sin(h\omega_0 + \varphi_h)$$
(29)

The voltage rms value can be written as:

$$V_{rms} = \sqrt{\frac{1}{T}} \int_{0}^{T} V^{2}(t) dt$$

$$= \sum_{h=2}^{\infty} V_{h}^{2} = V_{1}^{2} + V_{2}^{2}$$

$$+ V_{3}^{2} + V_{4}^{2} + \dots + V_{h}^{2}$$

$$I_{rms} = \sqrt{\frac{1}{T}} \int_{0}^{T} I^{2}(t) dt$$

$$= \sum_{h=2}^{\infty} I_{h}^{2} = I_{1}^{2}$$

$$+ I_{2}^{2} + I_{3}^{2} + I_{4}^{2} + \dots + I_{h}^{2}$$
(30)
(31)

The *r.m.s.* voltage or current for "total harmonic distortion" V_{thd} and I_{thd} , respectively, can be expressed as: Other simple but practical harmonic formulae include:

Total rms Current

$$\boldsymbol{I}_{rms} = \sqrt{\boldsymbol{I}_{fund}^2 + \boldsymbol{I}_{harm}^2}$$
(32)

or

$$\boldsymbol{I}_{rms} = \boldsymbol{I}_{find} \sqrt{1 + \left(\frac{\boldsymbol{I}_{thd}}{100}\right)^2}$$
(33)

Fundamental Current

$$\boldsymbol{I}_{fund} = \frac{\boldsymbol{I}_{rms}}{\sqrt{1 + \boldsymbol{I}_{hhd}^2}} \qquad (34)$$

2.1.1 Total Fundamental Current Distortion

$$I_{thd(fund)} = \sqrt{\left(\frac{I_{thd}}{I_{fund}}\right)^2 - 1}$$
(35)

2.1.2. Total Demand Distortion

$$\frac{\sqrt{\sum_{h=2}^{\infty} I_h^2}}{I_{load}} = I_{THD} = \frac{\sqrt{I_2^2 + I_3^2 + \dots + I_n^2}}{I_1}$$
(36)

Where:

 I_{load} = maximum demand load current (fundamental) at the PCC.

TDD = 'total demand distortion' of current (expressed as measured total harmonic current distortion, per unit of load current; for example, a 30% total current distortion measured against a 50% load would result in a TDD of 15%)

Study on the primary scope of harmonic modeling, simulation and steady-state distortion. The Fourier series for a regular, integrable, periodic function f(t), of period T seconds and fundamental frequency $\frac{f=1}{T}$ Hz or $\omega = 2 \pi f rad/s$, can be written as:

$$x(t) = a_0 + \sum_{n=1}^{\infty} - \sum_{n=1}^{\infty} b_n \sin * (2\pi f t n)$$
(37)

$$a_0 = \frac{I}{T} \int_{-T2}^{T2} x(t) dt \qquad (38)$$

$$a_n = \frac{2}{T} \int_{-T2}^{T2} x(t) \cos\left\{\frac{2\pi tn}{T}\right\} dt$$
 (39)

$$b_n = \frac{2}{T} \int_{-T_2}^{T_2} x(t) \sin\left\{\frac{2\pi tn}{T}\right\} dt \qquad (40)$$

The sine and cosine equation can be converted to the convenient polar form by using trigonometry as follows:

$$a_{n}Cos(k\omega_{1}t) + b_{n}Sin(k\omega_{1}t)$$

$$= \sqrt{a_{n}^{2} + b_{n}^{2}} * \frac{a_{n}Cos(k\omega_{1}t) + b_{n}Sin(k\omega_{1}t)}{\sqrt{a_{n}^{2} + b_{n}^{2}}}$$

$$= \sqrt{a_{n}^{2} + b_{n}^{2}} * \left\{ \frac{a_{n}}{\sqrt{a_{n}^{2} + b_{n}^{2}}} Cos(k\omega_{1}t)}{+ \frac{b_{n}}{\sqrt{a_{n}^{2} + b_{n}^{2}}}} Sin(k\omega_{1}t)} \right\}$$

$$= \sqrt{a_{n}^{2} + b_{n}^{2}} * \left\{ \frac{Sin(\theta_{n})Cos(k\omega_{1}t)}{+Cin(\theta_{n})Sos(k\omega_{1}t)}} \right\}$$

$$Sin(\theta_{n}) = \frac{a_{n}}{\sqrt{a_{n}^{2} + b_{n}^{2}}}$$

$$(41)$$

3. Conclusion

The work presents the investigation of winding dynamic deformations of an oil-immersed-type power transformer in a substation. The results reflect the effects of load and elastic properties on winding deformation, which can be useful for transformer design and fault diagnosis. The diagnostics methods for the inter-turn fault in transformers are employed in practice. The finite element model of a three-legged transformer is developed, which demonstrates the effect of inter-turn fault on the magnetic field intensity distribution inside the transformer. Modelling layer-type distribution transformers by representing the turns and layers by transmission lines has been performed.

References

- [1] R.S. Bhide et al., "Analysis of Winding Inter-turn Fault in Transformer: A Review and Transformer Models," *IEEE International Conference on Sustainable Energy Technologies*, pp. 1-7, 2010. [CrossRef] [Google Scholar] [Publisher Link]
- [2] Winding Conditions Using Regression Analysis and Frequency Response Measurements.
- [3] Marjan Popov et al., "Analysis of Very Fast Transients in Layer-Type Transformer Windings," *IEEE Transactions on Power Delivery*, vol. 22, no. 1, pp. 238-247, 2007. [CrossRef] [Google Scholar] [Publisher Link]
- [4] Bonginkosi A. Thango et al., "A Novel Approach to Assess Power Transformer Winding Conditions Using Regression Analysis and Frequency Response Measurements," *Energies*, vol. 15, no. 7, 2022. [CrossRef] [Google Scholar] [Publisher Link]