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Original Article

Applications of Dinesh Verma Transform to Beam Uniformly Loaded, One End Fixed and the Second End Subjected to Tensile Force

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Abstract **-** *The Laplace transform method is typically used to solve differential equations. The work investigates Dinesh Verma Transform differential equations. Dinesh Verma transformation makes it easier to solve differential problems in engineering applications and makes differential equations simple to solve. The goal of the paper is to demonstrate how Dinesh Verma transformation may be used to analyze differential equations.*

Keywords - *Dinesh Verma Transform, Differential equations.*

1. Introduction

Dinesh Verma Transform approach has been applied in solving boundary value problems in most of the science and engineering disciplines [1], [2], [3], [4], [5]. It also comes out to be a very effective tool for analyzing differential equations method [6], [7], [8], [9], [10]. The differential equations are generally solved by adopting Laplace transform method or convolution method of residue theorem [11], [12], [13], [14], [15]. The Dinesh Verma transformation is a mathematical tool used to solve differential equations by converting them from one form into another. Regularly it effectively solves linear differential equations, either ordinary or partial. The Dinesh Verma transformation is used in solving the time domain function by converting it into a frequency domain function. Dinesh Verma transformation makes it easier to solve the differential problem in engineering application and make differential equations simple to solve. This paper presents a new technique called Dinesh Verma transform to analyze differential equations.

2. Definitions

2.1. Basic Definition

Definition of Dinesh Verma Transform (DVT)

Dr. Dinesh Verma recently introduced a novel transform and named it Dinesh Verma Transform (DVT). Let f(t) be a well-defined function of real numbers t ≥ 0. The Dinesh Verma Transform (DVT) of f(t), denoted by $D\{\{f(t)\}$, is defined as [1]

$$D\{\{f(t)\} = p^5 \int_0^\infty e^{-pt} f(t)dt = \bar{f}(p)$$

Provided that the integral is convergent, where $p$may be a real or complex parameter and D is the Dinesh Verma Transform (DVT) operator.

Dinesh Verma transform of elementary functions:
According to the definition of Dinesh Verma Transform (DVT),

$$D\{t^n\} = p^5 \int_0^\infty e^{-pt} t^n dt$$

$$= p^5 \int_0^\infty e^{-z} \left(\frac{z}{p}\right)^n \frac{dz}{p} \quad, z = pt$$

$$= \frac{p^5}{p^{n+1}} \int_0^\infty e^{-z} (z)^n dz$$

Applying the definition of gamma function,

$$D\{y^n\} = \frac{p^5}{p^{n+1}} \lceil(n+1)$$

$$= \frac{1}{p^{n-4}} n!$$

$$= \frac{n!}{p^{n-4}}$$

Hence, $D\{t^n\} = \frac{\boldsymbol{n!}}{\boldsymbol{p^{n-4}}}$

Dinesh Verma Transform (DVT) of some elementary Functions

$$D\{t^n\} = \frac{n!}{p^{n-4}} \text{ , where } n = 0,1,2, ..$$

$$D\{e^{at}\} = \frac{p^5}{p-a},$$

$$D\{sinat\} = \frac{ap^5}{p^2+a^2},$$

$$D\{cosat\} = \frac{p^6}{p^2+a^2},$$

$$D\{sinhat\} = \frac{ap^5}{p^2-a^2},$$

$$D\{coshat\} = \frac{p^6}{p^2-a^2}.$$

$$D\{\delta(t)\} = p^5$$

The Inverse Dinesh Verma Transform (DVT) of some of the functions are given by

$$D^{-1}\left\{\frac{1}{p^{n-4}}\right\} = \frac{t^n}{n!} \text{ , where } n = 0,1,2, ..$$

$$D^{-1}\left\{\frac{p^5}{p-a}\right\} = e^{at},$$

$$D^{-1}\left\{\frac{p^5}{p^2+a^2}\right\} = \frac{sinat}{a},$$

$$D^{-1}\left\{\frac{p^6}{p^2+a^2}\right\} = cosat,$$

$$D^{-1}\left\{\frac{p^5}{p^2-a^2}\right\} = \frac{sinhat}{a},$$

$$D^{-1}\left\{\frac{p^6}{p^2-a^2}\right\} = coshat,$$

$$D^{-1}\{p^5\} = \delta(t)$$

Dinesh Verma Transform (DVT) of derivatives [1], [2], [10], [21].

$$D\{f'(t)\} = p\bar{f}(p) - p^5 f(0)$$

$$D\{f''(t)\} = p^2\bar{f}(p) - p^6 f(0) - p^5 f'(0)$$

$$D\{f'''(y)\} = p^3\bar{f}(p) - p^7 f(0) - p^6 f'(0) - p^5 f''(0)$$ And so on.

$$D\{tf(t)\} = \frac{5}{p}\bar{f}(p) - \frac{d\bar{f}(p)}{dp},$$

$$D\{tf'(\mathrm{t})\} = \frac{5}{p}\left[p\bar{f}(p) - p^5\mathrm{f}(0)\right] - \frac{d}{dp}\left[p\bar{f}(p) - p^5\mathrm{f}(0)\right]$$ and

$$D\{tf''(\mathrm{t})\} = \frac{5}{p}\left[p^2\bar{x}(p) - p^6 x(0) - p^5 x'(0)\right] - \frac{d}{dp}\left[p^2\bar{x}(p) - p^6 x(0) - p^5 x'(0)\right]$$ And so on.

and so on

3. Material and Method

- The equations of motion of a particle under certain conditions are

$$m\ddot{x} + e\mathrm{h}\dot{y} = eE \quad (1)$$

$$m\ddot{y} - e\mathrm{h}\dot{x} = 0 \quad (2)$$

with conditions

$x(0) = 0$, $x'(0) = 0$, $y(0) = 0$, $y'(0) = 0$

We will find the path of the particle at any instant.

Taking Dinesh VermaTransform of (1) on both sides

$$mD\{\ddot{x}\} + e\mathrm{h}D\{\dot{y}\} = D\{eE\}$$

Or

$$mp^2\bar{x}(p) - mp^6 x(0) - mp^5 x'(0) + e\mathrm{h}p\bar{y}(p) - e\mathrm{h}\, p^5\mathrm{y}(0) = eEp^4$$

Or

$$mp^2\bar{x}(p) + e\mathrm{h}p\bar{y}(p) = eEp^4 \quad (3)$$

Taking Dinesh Verma Transform of (2) on both sides

$$mD\{\ddot{y}\} - e\mathrm{h}\mathrm{D}\{\dot{x}\} = 0$$

Or

$$mp^2\bar{y}(p) - mp^6 y(0) - mp^5 y'(0) - e\mathrm{h}p\bar{x}(p) + e\mathrm{h}\, p^5 x(0) = 0$$

Or

$$mp^2\bar{y}(p) - e\mathrm{h}p\bar{x}(p) = 0 \quad (4)$$

Solving (3) & (4), we get,

$$\bar{x}(p) = meE\left\{\frac{p^4 h}{p^2m^2 + e^2h^2}\right\}$$

Or

$$\bar{x}(p) = \left\{\frac{p^4\frac{eh}{m}m^2}{p^2m^2 + e^2h^2}\right\}$$

$$\bar{x}(p)=\left\{\frac{p^4\,Ew}{(p^2+w^2)h}\right\}$$

$$\bar{x}(p)=\frac{E}{hw}\left\{p^4-\frac{p^6}{(p^2+w^2)}\right\}$$

$$where\ w=\frac{eh}{m}$$

Or

Taking inverse Dinesh Verma transform,

$$x=\frac{E}{\text{hw}}\,[1-coswt]$$

And,

$$\bar{y}(p)=\left\{\frac{e^2E\text{h}p^3}{m^2p^2+e^2h^2}\right\}$$

$$\bar{y}(p)=\left\{\frac{w^2E\text{m}p^3}{(p^2+w^2)eh^2}\right\},where\ w=\frac{eh}{m}$$

$$\bar{y}(p)=\frac{Em}{eh^2}\left\{\frac{wp^5+w^3p^3-wp^5}{(p^2+w^2)}\right\}$$

$$\bar{y}(p)=\frac{E}{hw}\left\{wp^3-\frac{wp^5}{w^2+p^2}\right\}$$

Or

Taking inverse Dinesh Verma transform,

Or

$$y=\frac{E}{\text{hw}}\{wt-sinwt\}$$

- The differential equation satisfied by a beam uniformly loaded, one end fixed and the second end subjected to tensile force P, is given by

$E.I.\ddot{y}=Py-\frac{1}{2}Wt^2=0$, with conditions $y(0)=0\,,y'(0)=0.$

$$We\ will\ find\ the\ deflection\ at\ any\ length\ of\ the\ beam.$$

We have

$$E.I.\ddot{y}=Py-\frac{1}{2}Wt^2=0$$

This equation can be written as

$$\ddot{y}-\frac{P}{EI}y=\frac{W}{2EI}t^2$$

Taking Dinesh Verma Transform on both sides

$$D\{\ddot{y}\}-\frac{P}{EI}D\{y\}=-\frac{W}{2EI}D\{t^2\}$$

$$p^2\bar{y}(p)-p^6y(0)-p^5y^{'}(0)-\frac{P}{EI}\bar{y}(p)=-\frac{W}{2EI}2p^2$$

Or

$$p^2\bar{y}(p)-\frac{P}{EI}\bar{y}(p)=-\frac{W}{2EI}2p^2$$

Or

$$\left[p^2-\frac{P}{EI}\right]\bar{y}(p)=-\frac{W}{EI}p^2$$

or

$$\bar{y}(p)=-\frac{Wp^2}{(EIp^2-P)}$$

On simplifying and taking inverse Dinesh Verma transform,

$$y=-W\left[-\frac{t^2}{2P}-\frac{EI}{P^2}+\frac{EI}{P^2}coshnt\right]$$

or

$$where\ n^2=\frac{P}{EI}$$

Or

$$y=\left[\frac{Wt^2}{2P}-\frac{EI}{P^2}+\frac{W}{Pn^2}[1-coshnt]\right]$$

4. Conclusion

In this study, we successfully used the Dinesh VermaTransform approach to analyze differential equations. It is demonstrated that the method is effective when analyzing differential equations.

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