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Original Article

Robust Virtual Sensors Design Based on Jordan Canonical Form

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Abstract - *The problem of robust virtual sensor design in linear systems under disturbance is studied. The problem is solved in two different ways: in the first way, the linear model insensitive to the disturbance is designed and then used as a virtual sensor; in the second one, the model has minimal sensitivity to the disturbance, and the sliding mode observer is constructed. The relations allowing to design virtual sensor of minimal dimension estimating given component of the system's state vector are obtained. The well-known example of the three-tank system illustrates the theoretical results.*

Keywords - *Linear systems, Virtual sensors, Reduced order models, Sliding mode observers, Canonical form.*

1. Introduction

To solve the problems of control and fault diagnosis, modern complex technical systems have different physical sensors. Clearly, the more sensors are measured, the simpler solution to the problems is obtained. Additional physical sensors may result in extra expenses; besides, they are not high reliability. Virtual sensors can help in this case.

Many papers are considering different problems in designing and applying virtual sensors. Most of these papers consider different practical applications of virtual sensors: for monitoring automotive engines [1], for active reduction of different noises in active control systems [3], for hiding the fault from the controller's point of view [9], to construct walking legged robots [10], for diagnosis in different systems: in aircraft [11], in industrial motors [12], in mixing machine [13], in the sensor-cloud platform [17], for a tunnel furnace [24]. A new architectural paradigm for remotely deployed sensors is presented in [21]. Theoretical aspects of using virtual sensors in linear systems are considered in [2, 16]; in [23], virtual sensors are used for fault tolerant control in linear descriptor systems. The procedure to design virtual sensors for systems described by linear models is suggested in [4]. Note that virtual sensors in [4] estimate the system state vector and are of the whole dimension.

The main contribution of the present paper is that we design virtual sensors of minimal dimension for systems described by linear models under the disturbance estimating prescribed elements of the vector of state invariant with respect to or having minimal sensitivity to the disturbance. The set of the prescribed components depends on the problem of control or fault diagnosis under consideration. In particular, the virtual sensor can be used to replace the faulty physical sensor in the control system to continue the operation.

The rest of the paper is organized as follows. In Section 2, the main models are introduced. Section 3 solves the problem of the reduced-order model design. In Section 4, a Sliding Mode Observer (SMO) is constructed. An example of a three-tank system is considered in Section 5. Section 6 concludes the paper.

2. Preliminaries

Consider the system described by the linear dynamic model.

$$\dot{x}(t) = Ax(t)+Bu(t)+Dd(t),\\ y(t) = Cx(t), \quad (1)$$

Where $x(t) \in R^n$, $y(t) \in R^l$, and $u(t) \in R^m$ are vectors of state, control and output, A, B, C, and D are known constant matrices, $d(t) \in R^p$ is the disturbance, we assume that $d(t)$ is an unknown bounded function of time: $||d(t)|| \le d_*$.

The problem under consideration is as follows: construct a virtual sensor of minimal dimension estimating the variable $z(t) = Mx(t) \in R$ for a given matrix M , invariant with respect to the disturbance or having minimal sensitivity to it. To solve the problem, one uses the reduced order model of the original system, which is invariant with respect to the disturbance or has minimal sensitivity to it. Based on this model, the virtual sensor estimating the variable $z(t)$ is designed.

We consider two different kinds of the model: the first one estimates the variable $z(t)$, and the second estimates the same variable $y_*(t) = R_*x(t)$ for some matrix R_*. The first model can be used to design the virtual sensor without additional constructions; the second one assumes to construct SMO. The second way is more complex than the first, but SMO allows finite-time convergence.

The first model invariant with respect to the disturbance is described as follows:

$$\begin{aligned} \dot{x}_*(t) &= A_*x_*(t)+B_*u(t)+J_0y_0(t), \\ z(t) &= C_zx_*(t)+Qy(t), \end{aligned} \tag{2}$$

where $x_*(t) \in R^k$, $k < n$, is the state vector, A_*, B_*, J_0, C_z, and Q are matrices that should be determined,

$$y_0(t) = C_0x(t) = \begin{pmatrix} C \\ M \end{pmatrix} x(t) = \begin{pmatrix} y(t) \\ z(t) \end{pmatrix}.$$

The second model invariant with respect to the disturbance is given by

$$\begin{aligned} \dot{x}_*(t) &= A_*x_*(t)+B_*u(t)+J_0y_0(t), \\ y_*(t) &= C_*x_*(t). \end{aligned} \tag{3}$$

To solve such a problem, the canonical identification form of the matrix A_* is used. Unlike in this paper, the matrix A_* is in Jordan's canonical form.

$$A_* = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_k \end{pmatrix}, \tag{4}$$

Where $\lambda_1,\ldots,\lambda_k$ are eigenvalues of the matrix A_*. This allows us to simplify a solution and reduce the dimension of the virtual sensor.

If the model invariant for the disturbance cannot be constructed, the model with minimal sensitivity to the disturbance is designed. In this case, the variable z(t) is estimated by SMO based on such a model.

3. Reduced Order Model Design

3.1. The Main Relations

To design both models, introduce the matrix Ψ such that $x_*(t) = \Psi x(t)$. It is known that matrices describing the models (2) and (3) satisfy the conditions.

$$\begin{aligned} \Psi A &= A_*\Psi+J_0C_0, \\ B_* &= \Psi B, \ \Psi D = 0 \end{aligned} \tag{5}$$

For the first model, the additional condition appears because of the equation $z(t) = Mx(t) = C_zx_*(t)+Qy(t)$; since $x_*(t) = \Psi x(t)$ and $y(t) = Cx(t)$, it follows:

$$M = C_z\Psi+QC. \tag{6}$$

Rewrite this relation in the form

$$M = C_z\Psi+QC = (C_z \quad Q)\begin{pmatrix} \Psi \\ C \end{pmatrix}. \tag{7}$$

It is satisfied if and only if

$$\operatorname{rank}\begin{pmatrix} \Psi \\ C \end{pmatrix} = \operatorname{rank}\begin{pmatrix} \Psi \\ C \\ M \end{pmatrix}. \tag{8}$$

If it is true, the variable $z(t)$ can be estimated by the observer.

For the second model, the additional condition is of the form $R_*C = C_*\Psi$. Rewrite it in the form

$$(R_* \quad -C_*)\begin{pmatrix} C \\ \Psi \end{pmatrix} = 0 \tag{9}$$

This equation has a solution if and only if

$$\operatorname{rank}\begin{pmatrix} \Psi \\ C \end{pmatrix} < \operatorname{rank}(\Psi) + \operatorname{rank}(C). \tag{10}$$

An additional restriction of insensitivity to the disturbance $\Psi D = 0$ can be considered as follows. Let D_0 be the matrix of maxima rank such that $D_0D = 0$; as a result, $\Psi = ND_0$ with some matrix N. In this case, the second equation in (5) can be rewritten in the form

$$(N_i \quad -J_{*i})\begin{pmatrix} D_0(A-\lambda_iI_n) \\ C \end{pmatrix} = 0, \qquad i=\overline{1,k} \tag{11}$$

Where I_n is the identity matrix.

To solve equation (11), one has to choose $\lambda_i < 0$, i=1,2,...k, then find from $\Psi = ND_0$ the minimum number of rows $\Psi_i = N_iD_0$ satisfying the condition (8), and finally, calculating the matrices C_z and Q from (7). Since $\lambda_i < 0$, the model is stable and can be used as the virtual sensor.

If condition (8) is not satisfied for all $\Psi_i = N_iD_0$, the first model invariant to the disturbance does not exist, and one has to design the second model by checking the condition (10). If it is satisfied, the matrices R_* and C_* are found in (9), and the second model can be designed.

If (11) has no solutions with $\lambda_i < 0$, one has to use a robust method.

3.2. Robust Model Design

The contribution of the disturbance in (2) and (3) is estimated by the norm $||\Psi D||_F$ of the matrix ΨD. To minimize this norm, choose $\lambda_i < 0$ for which (11) with $D_0 = I$ is solvable and find all solutions in the form $\Psi_i^{(1)}, \ldots, \Psi_i^{(N)}$. Represent these solutions in the form.

$$\Psi_{*i}=\begin{pmatrix}\Psi_i^{(1)}\\ \vdots \\ \Psi_i^{(N)}\end{pmatrix}, \quad J_i=\begin{pmatrix}J_{0i}^{(1)}\\ \vdots \\ J_{0i}^{(N)}\end{pmatrix}.$$

To solve the problem, find the singular value decomposition of the matrix $\Psi_{*i}D$:

$$\Psi_{*i}D = U_D \Sigma_D V_D,$$

where U_D and V_D are orthogonal matrices,

$$\Sigma_D = (\mathrm{diag}(\sigma_1,\ldots,\sigma_c)\ \ 0) \text{ or } \Sigma_D = \begin{pmatrix}\mathrm{diag}(\sigma_1,\ldots,\sigma_c)\\ 0\end{pmatrix},$$

$c = \min(N,kp)$, $0 \le \sigma_1 \le \ldots \le \sigma_c$ are singular values of the matrix $\Psi_{*i}D$ [15]. Choose the first transposed column of the matrix U_D as a vector $w = (w_1,\ldots,w_N)$, calculate the matrices $\Psi_i = w\Psi_{*i}$, $J_{0i}=wJ_i$, $i=1,2,\ldots k$, and find the matrices.

$$\Psi=\begin{pmatrix}\Psi_1\\ \vdots \\ \Psi_k\end{pmatrix}, \quad J_0=\begin{pmatrix}J_{01}\\ \vdots \\ J_{0k}\end{pmatrix}.$$

Then the possibility of estimating the variable $z(t)$ is checked based on (8). If it is satisfied, a model with minimal sensitivity to the disturbance can be designed. Otherwise, one finds another vector w related to the singular value greater than σ_1.

Theorem 1. The vector $w = (w_1,\ldots,w_N)$ produces the optimal solution with the minimal norm of the matrix ΨD.

Proof. It follows from the properties of singular value decomposition [15].

If condition (8) is true with the matrix Ψ for some k, find the matrices H_z and Q from (7) and set $B_* = \Psi B$, $D_* = \Psi D$. As a result, the virtual sensor with minimal sensitivity to the disturbance estimating the variable $z(t)$ has been designed
Assume that (8) is not true for all k. In this case, the model

$$\begin{aligned}\dot{x}_*(t) &= A_* x_*(t)+B_* u(t)+J_* d(t)+ J_z z(t),\\ y_*(t) &= C_* x_*(t),\end{aligned} \tag{12}$$

where $(J_* \ \ J_z) = J_0$ will be used to design a sliding mode observer estimating the variable $z(t)$.

4. Sliding Mode Observer Design

If $J_z \ne 0$, we may begin to design the observer. Otherwise, the model (12) should be transformed as follows:

$$\begin{aligned}\dot{x}_*(t) &= A_* x_*(t) + B_* u(t) + J_* d(t) + J_z z(t)\\ &\quad + P_0(C_z x_*(t) + Qy(t) - C_z x_*(t) - Qy(t))\\ &= A_*' x_v(t) + B_* u(t) + J_*' y(t) + D_0 z(t),\\ y_*(t) &= C_* x_*(t),\end{aligned}$$

where $A_*' = A_* - P_0 C_z$, $J_*' = J_* - P_0 Q$, the matrix P_0 can be chosen in the form $P_0=(0 \ \ 0 \ \ \cdots \ \ 1)^T$. It is assumed additionally that $z(t)$ is the bounded function of time and $||z(t)|| \le \beta$ for some positive β.

Remark. If $J_z \ne 0$, the above transformation is not necessary, and we set $J_*' = J_*$ and $A_*'=A_*$. Assume for simplicity that $J_z \ne 0$.

Different methods to design SMO exist [5-7, 18-20, 25, 26]. We will use the one suggested in [23]. To implement the method [23], assume that $k = 1$ and $A_* = -\lambda_1$ for some $\lambda_1 > 0$. The model (12) is in the form

$$\begin{aligned}\dot{x}_*(t) &= -\lambda_1 x_*(t) + J_* y(t) + B_* u(t) + J_z z(t),\\ y_*(t) &= x_*(t),\end{aligned} \tag{13}$$

SMO is described by

$$\begin{aligned}\dot{\hat{x}}_*(t) &= \lambda_1 \hat{x}_*(t) + J_* y(t) + B_* u(t) - k_1 v(t),\\ \hat{y}_*(t) &= \hat{x}_*(t),\end{aligned} \tag{14}$$

Where $v(t)=\mathrm{sign}(e(t))$, $e(t) = \hat{y}_*(t) - R_* y(t)$, $k_1 > 0$.

It follows from (13) and (14) that the estimation error $e(t)$ is given by

$$\dot{e}(t) = \lambda_1 e(t) - k_1 v(t) - J_z z(t). \tag{15}$$

Because $z(t)$ is bounded function and $|v(t)| = 1$, then $|k_1 v(t) + J_z z(t)| \le g_0$ for some $g_0 > 0$. It can be shown that $e(t)$ is bounded as well and $|e(t)| \le \delta$ for some $\delta > 0$.

Theorem 2. The function $z(t)$ is estimated by the observer (14) as

$$\hat{z}(t) = -J_z^{-1} k_1 v_e(t). \tag{16}$$

Here $v_e(t)$ is the signal representing the average behavior of $v(t)$. According to [4], one uses as $v_e(t)$ the function

$$v_e(t) = e(t)/(|e(t)| + \varepsilon)$$

with a small positive scalar ε.

Proof. One proves that $e = 0$ in finite time by choice of observer gain k_1, therefore, sliding motion is achieved. Introduce Lyapunov candidate function $V(t) = e^2(t)$ and find its derivative to time taking into account (15):

$$\dot{V}(t)=2e(t)\dot{e}(t) = 2e(t)(\lambda_1 e(t) - k_1 v(t) - J_z z(t)).$$

Since $v(t) = \mathrm{sign}(e(t))$, then $e(t)k_1 v(t) = k_1|e(t)|$ and

$$\dot{V}(t) \le 2|e(t)|(\lambda_1\delta - k_1 - \beta||J_z||).$$

If $k_1 > \lambda_1\delta - k_1 - \beta||J_z||$, then $\dot{V}(t) < 0$, therefore the sliding motion is achieved, that is, $e(t) = \dot{e}(t) = 0$ in finite time. Then it follows from (15) that the fault is estimated by (16).

Note that if measurement noise $w(t) \ne 0$ is present in the form $y(t) = Cx(t) + w(t)$ and $||w(t)|| \le w_*$, the main result remains as before, but the demand for the coefficient k_1 becomes more rigorous. In this case, equation (16) is supplemented by the term $J_* w(t)$:

$$\dot{e}(t) = \lambda_1 e(t) - k_1 v(t) - J_z z(t) - J_* w(t).$$

Then the additional term appears in the derivative $\dot{V}(t)$:

$$\dot{V}(t) \le 2|e(t)|(\lambda_1\delta - k_1 + \beta||J_z|| + w_*||J_*||);$$

formula for k_1 becomes:

$$k_1 > \lambda_1\delta + \beta||J_z|| + w_*||J_*||.$$

The estimate (16) becomes approximate:

$$\hat{z}(t) \approx -J_z^{-1} k_1 v_e(t).$$

Note that due to the singular value decomposition contribution of the term $D_* d(t)$ in the estimation $\hat{z}(t)$ is as minimal as possible.

5. Example

Consider the control system

$$\begin{aligned}
\dot{x}_1(t) &= u_1(t) - b_1(x_1(t) - x_2(t)),\\
\dot{x}_2(t) &= u_2(t) + b_1(x_1(t) - x_2(t)) - b_2(x_2(t) - x_3(t)) + d_1(t),\\
\dot{x}_3(t) &= b_2(x_2(t) - x_3(t)) - b_3(x_3(t) - \vartheta_3) + d_2(t),\\
y_1(t) &= x_2(t), \quad y_2(t) = x_3(t).
\end{aligned}$$

The equations represent a model of the well-known three-tank system. The system consists of three consecutively united tanks. Pipes link the tanks. The liquid flows into the first and the second tanks and follows from the third tank through the pipe. The levels of liquid in the tanks are x_1, x_2, and x_3, respectively. For simplicity, it is assumed that $a_1 = ... = a_6 = 1$.

The system is described by the following matrices

$$A=\begin{pmatrix}-1 & 1 & 0\\ 1 & -2 & 1\\ 0 & 1 & -2\end{pmatrix},\quad B=\begin{pmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{pmatrix},$$

$$C=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix},\quad B=\begin{pmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{pmatrix},$$

The problem is to estimate the variable $x_1(t)$. Since $D_0 = (1 \quad 0 \quad 0)$, equation (11) becomes

$$(N_i \; -J_{*i})\begin{pmatrix}-1-\lambda_i & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} = 0$$

and has a solution with $\lambda_1 = -1$: $N_1 = 1$, $J_{*1} = (1 \quad 0)$, $\Psi = (1 \quad 0 \quad 0)$. The model invariant with respect to the disturbance is given by

$$\dot{x}_*(t) = -x_*(t) + y_1(t).$$

Since this model is stable, it can be used as a virtual sensor.

Note that in [28] where the canonical identification form of the matrix A_* is used, the virtual sensor is 2-dimensional and is sensitive to disturbance. The example demonstrates the advantage of Jordan's canonical form.

For simulation, consider a three-tank system and the observer with the controls $u_1(t) = 1, t \ge 1$ and $u_2(t) = 0.5, t \ge 5$; $d_1(t) = -0.3$, $t \ge 6$; $d_2(t) = -0.4$, $t \ge 10$; $\varepsilon = 0.1$. Simulation results are shown in Figure. 1, where the functions $x_1(t)$ and $z(t)$ are presented.

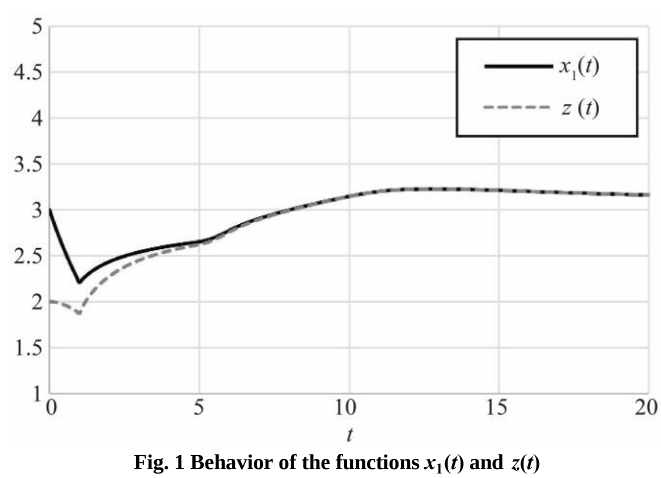

Fig. 1 Behavior of the functions $x_1(t)$ and $z(t)$

6. Conclusion

In this paper, the problem of virtual sensor design has been considered for linear dynamic systems under disturbance. The suggested approach enables obtaining a virtual sensor of minimal dimension based on the reduced order model of the original system invariant to or having minimal sensitivity to the disturbance. This allows the extension of a class of systems for which virtual sensors can be designed. Two kinds of models have been suggested: the first model can be used to design the virtual sensor without additional constructions, and the second one assumes to construct SMO. A sliding mode observer is used to construct the virtual sensor. The future research direction is the virtual sensor design for hybrid nonlinear dynamic systems.

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