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LDPC Codes Performance using Max-Log-MAP Algorithm for High Order Constellations

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Abstract
The exact calculation, LLR (Log-Likelihood Ratio) or APP (A Posteriori Probability) of the QAM (Quadrature Amplitude Modulation) demapping involves complicated operations. Several algorithms have been introduced to simplify the calculation of the LLR, such as a pragmatic algorithm, Max-Log-MAP algorithm. In this work, we apply the Max-Log-MAP algorithm for binary LDPC codes. Simulation results show that the LDPC codes using the Max-Log-MAP algorithm for 16-QAM and 64-QAM constellations can achieve a good performance of binary LDPC codes with a simple calculation over Gaussian and Rayleigh channels.

Keywords **—** *LDPC codes, LLR, Max-log-Map algorithm, Pragmatic algorithm, QAM mapping, Sum-Product Algorithm.*

I. INTRODUCTION

High-order constellations can achieve enhanced high-speed transmission without increasing bandwidth [1]. For this reason, Quadrature Amplitude Modulation (QAM), which has been adopted by various communication standards, is strongly recommended as a high order constellation. However, communication systems using the MAQ require a high signal-to-noise ratio. So it is advantageous to combine with the MAQ efficient error-correcting codes [2,3], such as LDPC codes. LDPC codes [4,5] are error-correcting codes and can approach the Shannon limit for large data blocks [6]. They are block codes with parity-check matrices H that contain only a minimal number of non-zero entries. This sparseness of H is essential for an iterative decoding complexity that increases only linearly with the code length. The LDPC codes can be described by a graphical representation called Tanner graph [7], which corresponds to the matrix H.

Tanner graph is a bipartite graph composed of two types of nodes: the variable nodes representing the symbols of the codeword and the parity nodes representing the parity control equations. Branches connect these two types of nodes according to the non-zero elements in the parity check matrix. Each node generates and propagates messages to its neighbours based on its current incoming messages except the input message on the branches where the output message sent.

LDPC codes are decoded iteratively using a graphical representation of their parity-check matrix. The first iterative decoding algorithm of the LDPC codes is the Sum-Product Algorithm (SPA) [4,5], also known as the belief propagation algorithm, which is an optimal iterative decoding algorithm but with high computational complexity. Several algorithms have been proposed to reduce the complexity of the SPA [8].

The LDPC decoder must operate on soft decisions calculated using: LLR (Log-Likelihood Ratio) or APP (A Posteriori Probability) according to the type of decoding algorithm used. The exact calculation of these decisions for higher-order constellations involves complicated operations. Several algorithms have been introduced to simplify the calculation of the LLR for the binary codes, such as the pragmatic algorithm [9], The max-log-MAP (Maximum A Posteriori) algorithm [10], and the simplified max-log-MAP [11]. The pragmatic algorithm is studied for binary and non-binary LDPC codes, respectively, in [20] and [13]. In this work, we use the algorithm max log-MAP to simplify the LLR calculation for binary LDPC codes.

The rest of the paper is organized as follows. Section 2 introduces the LDPC decoding: SPA that was used in the simulation. In Sections 3 and 4, the exact LLR computation and the Max-Log-MAP algorithm are investigated, respectively. Finally, the simulation results and concluding remarks are given in Sections 4 and 5, respectively.

II. LDPC DECODING: SUM-PRODUCT ALGORITHM

In the following, the SPA is described.

➢ *Sum-Product algorithm*

The SPA performs the following operations [14]:

- Initialization of variable nodes

$$\mu_{mn} = log\frac{Pr(v_n=1|c'_n)}{Pr(v_n=0|c'_n)}, m\epsilon\{1,\dots,M\},n\epsilon\{1,\dots,N\}(1)$$

- Iteration

- Parity check nodes computation

$$\beta_{mn} = 2\times\tanh^{-1}\left(\prod_{n'\in N_m/n}\tanh\left(\mu_{mn'}/2\right)\right) \quad (2)$$

- Variable nodes computation

$$\mu_{mn} = \gamma_n + \sum_{m'\in M_n/m}\beta_{m'n} \quad (3)$$

- A posteriori information

$$\tilde{\gamma} = \gamma + \sum_{m\in M_n}\beta_{mn} \quad (4)$$

- Decision

$$\hat{c}_n = \begin{cases}0 & si\tilde{\gamma}_n > 0\\ 1 & si\tilde{\gamma}_n < 0\end{cases} \quad (5)$$

Finally, the algorithm stops if the maximum number of iterations is reached or if the syndrome is zero.

III. EXACT LLR COMPUTATION

2^{2m}-QAM transmit, at each time, 2^{2m} binary symbols. Each set of $2m$ binary symbols is associated with a symbol $c = a + jb$, where $a\ and\ b \in \{\pm1,\pm3,\pm5,\dots,2m\pm1\}$. After passing through the transmission channel, the observation relating to the symbol c is represented by the symbols $c' = a' + jb'$. At the reception, 2^{2m}-QAM-Gray demapping treat each symbols c' representative of the symbols c to extract $2m$ samples $\{\hat{u}_{n,i}\}, i \in \{1,\dots,2m\}$ each representative of a binary symbol $u_{n,i}$. The sample $\hat{u}_{n,i}$, the soft output demapping, is obtained using two relationships, $LLR(u_{n,i})$ (Log-Likelihood Ratio) or $APP(u_{n,i})$ (A Posteriori Probability). In this work, one used LLR computation. $LLR(u_{n,i})$, $i \in \{1,\dots,m\}$, is calculated as follows [15]:

$$LLR(u_{n,i}) = log\left[\frac{Pr\{(a'_n,b'_n)/u_{n,i}=1\}}{Pr\{(a'_n,b'_n)/u_{n,i}=0\}}\right] \quad (6)$$

Where $Pr\{(a'_n,b'_n)/u_{n,i} = w\}$ is the probability that the available couple is (a'_n,b'_n); knowing the binary symbol $u_{n,i}$ is equal to w.

For a square constellation $m = 2p$, 2^{2p}-QAM has the particularity to be reduced to two amplitude modulations with 2^P states independently acting on two carriers in-phase and quadrature [11]. According to this property (the case of a square constellation):

- The p expressions in phase are consequently the following:

$$LLR(u_{n,i}) = log\left[\frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n-a^0_{i,j}\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n-a^1_{i,j}\right)^2\right\}}\right] i \in \{1,\dots,p\}(7)$$

With $a^k_{i,j}$ are possible values of the symbol a_n when the symbol $u_{n,i}$ to be transmitted has the value k (k = 0 or 1); $w = 0\ or\ 1$;

- The p relations in quadrature eventually lead to the following expressions:

$$LLR(u_{n,i}) = log\left[\frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n-\alpha_n b^0_{i,j}\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n-\alpha_n b^1_{i,j}\right)^2\right\}}\right] i \in \{p+1,\dots,2p\}(8)$$

With $b^k_{i,j}$ are possible values of the symbol b_n when the symbol $u_{n,i}$ to be transmitted has the value k (k = 0 or 1).

Equations (7) and (8) are the exact calculation of the LLR; it is the optimal calculation that represents the log-MAP algorithm [16-18]. However, it involves several operations. Several algorithms have been introduced to simplify the exact calculation of the LLR. In this work, we use two simplified algorithms: a max-log-MAP algorithm and a simplified max-log-MAP algorithm.

IV. SIMPLIFIEDLLR COMPUTATION (MAX-LOG-MAPALGORITHM)

The exact calculation of the LLR involves several operations. Several algorithms have been introduced to simplify the exact calculation of the LLR. In this work, we use the max-log-MAP algorithms. The LLR is simplified as follows [21]:

$$LLR(u_{n,i}) = \frac{\max\limits_{j\in\{1,\dots,2^{p-1}\}} exp\left(\frac{1}{2\sigma^2}\left(a'_n-\alpha_n a^0_{i,j}\right)\right)^2}{\max\limits_{j\in\{1,\dots,2^{p-1}\}} exp\left(\frac{1}{2\sigma^2}\left(a'_n-\alpha_n a^1_{i,j}\right)\right)^2}, (9)$$

Where $i \in \{1,\dots,p\}$
And

$$LLR(u_{n,i}) = \frac{\max\limits_{j\in\{1,\dots,2^{p-1}\}} exp\left(\frac{1}{2\sigma^2}\left(b'_n-\alpha_n b^0_{i,j}\right)\right)^2}{\max\limits_{j\in\{1,\dots,2^{p-1}\}} exp\left(\frac{1}{2\sigma^2}\left(b'_n-\alpha_n b^1_{i,j}\right)\right)^2}, \quad (10)$$

Where $i \in \{p+1,\dots,2\ p\}$

V. SIMULATION RESULTS

In this section, we present the effect of the LLR simplified calculation on the performance of a binary LDPC code, with a code rate of 1/2 and a parity check matrix of size 512 × 1024, and for a decoding algorithm using the LLR at its input: the SPA algorithm. The

LDPC code is associated with two square constellations: 16-QAM and 64-QAM and the associated Gray mapping, and Gaussian and Rayleigh channels.

Figures 1 and 2 show respectively for 16-QAM and 64-QAM, performance comparisons, on a Gaussian channel, between an LDPC code using the exact calculation of the LLR and an LDPC code the simplified calculation by applying the max-log-MAP algorithm.

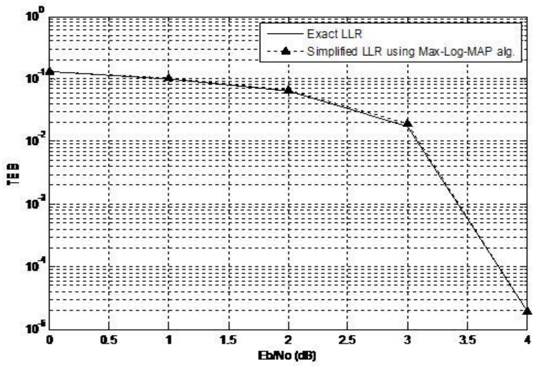

Fig. 1. Performance comparisons, under Gaussian channel, of (512, 1024) LDPC code using exact LLR computation and its simplified calculation using Max-Log-MAP algorithm, for 16-QAM

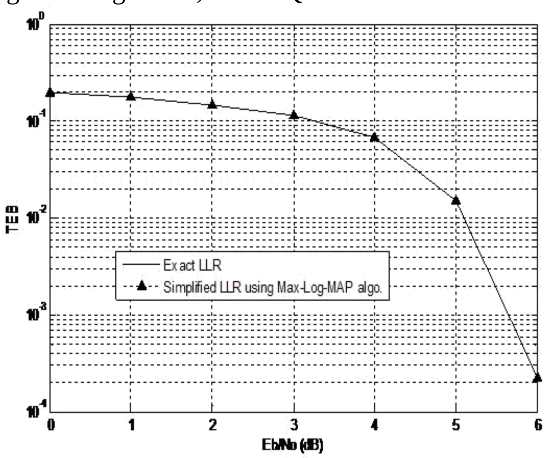

Fig. 2. Performance comparisons, under Gaussian channel, of (512, 1024) LDPC code using exact LLR computation and its simplified calculation using Max-Log-MAP algorithm, for 64-QAM

In figures 1 and 2, we can see that the LDPC code using the simplified LLR computations has a minimal performance loss for 16-QAM. For 64-QAM, there is no performance degradation. The same performance comparison obtained on a Gaussian channel is performed on a Rayleigh channel for 16-QAM (figure 3).

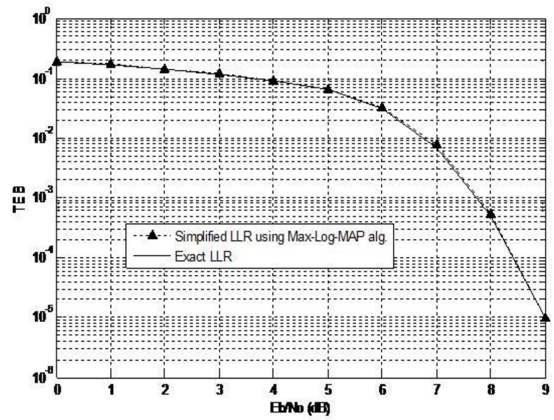

Fig. 3. Performance comparisons, under Rayleigh channel, of (512, 1024) LDPC code using exact LLR computation and its simplified calculation using Max-Log-MAP algorithm, for 16-QAM

In figures 3, we can see that the LDPC code using the simplified LLR computations has a minimal performance loss. As a result, the simplification of LLR calculation using the Max-Log-MAP algorithm can achieve a good performance of binary LDPC codes with a simple calculation.

VI. CONCLUSION

In this work, we have used for binary LDPC codes the simplified calculation of the LLR using the Max-Log-MAP algorithm. Simulation results show that the LDPC codes have a good performance with a simple calculation of LLR.

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