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Binary LDPC Codes Performance using Simplified Max-Log-MAP Algorithm for High Order Constellations over Gaussian Channel

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Abstract

Combining LDPC codes with high order constellations is an effective way to improve bandwidth efficiency. Since the LDPC codes require soft decisions regarding the input information, the constellation has to provide soft information (log-likelihood ratio) to it. In this paper, a simplified algorithm, called Simplified Max-Log-MAP algorithm, to calculate LLR is applied for binary LDPC codes, which remarkably reduce the computational complexity, and has nearly no performance loss.

Keywords—*LDPC codes, LLR, Max-log-Map algorithm, Pragmatic algorithm, QAM mapping, simplified Sum-Product Algorithm.*

I. INTRODUCTION

High-order constellations can achieve enhanced high-speed transmission without increasing bandwidth [1]. For this reason, Quadrature Amplitude Modulation (QAM), which has been adopted by various communication standards, is strongly recommended as a high order constellation. However, communication systems using the MAQ require a high signal-to-noise ratio. Hence, it is advantageous to combine with the MAQ efficient error-correcting codes [2, 3], such as LDPC codes.

LDPC codes [4, 5], are error-correcting codes, have a capability approaching the Shannon limit for large data blocks [6]. They are block codes with parity-check matrices *H* that contain only a minimal number of non-zero entries. This sparseness of *H* is essential for an iterative decoding complexity that increases only linearly with the code length. LDPC codes are decoded iteratively using a graphical representation of their parity-check matrix.

The first iterative decoding algorithm of the LDPC codes is the Sum-Product Algorithm (SPA) [4,5], also known as the belief propagation algorithm, which is an optimal iterative decoding algorithm, but with high computational complexity. Several algorithms have been proposed to reduce the complexity of the SPA [7].

The LDPC decoder must operate on soft decisions calculated using: LLR (Log-Likelihood Ratio) or APP (A Posteriori Probability) according to the type of decoding algorithm used. The exact calculation of these decisions for higher-order constellations involves complicated operations. Several algorithms have been introduced to simplify the analysis of the LLR for the binary codes such as the pragmatic algorithm [8], the max-log-MAP (Maximum A Posteriori) algorithm [9], and the simplified max-log-MAP [10]. The pragmatic algorithm is studied for binary and non-binary LDPC codes, respectively, in [11] and [12]. In this work, we use the algorithms max-log-MAP and simplified max-log-MAP to simplify the LLR calculation for binary LDPC codes.

The rest of the paper is organized as follows. Section 2 introduces the LDPC code and the SPA that used in the simulation. In Sections3 and 4, the exact calculations of LLR and Simplified Max-Log-MAP for QAM constellation under the Gaussian channel are investigated, respectively. Finally, the simulation results and concluding remarks are given in Sections4 and 5, respectively.

II. LDPC CODING AND DECODING

LDPC codes are linear block codes based on low-density parity-check matrices *H*, i.e., the number of non-zero elements in the matrix is much less than the number of zeros. The LDPC codes can be described by a graphical representation called Tanner graph [13], which corresponds to the matrix *H*.

Tanner graph is a bipartite graph composed of two types of nodes: the variable nodesv_n, $n\epsilon\{1,\dots,N\}$, representing the symbols of the codeword and the parity nodes nodes c_m, $m\epsilon\{1,\dots,M\}$, representing the parity control equations. Branches connect these two types of nodes according to the non-zero elements in the parity check matrix. Each node generates and propagates messages to its neighbours based on its current incoming messages except the input message on the branch where the output message sent.

The parity check matrix *H* allowed us to determine the Tanner graph, which is used as a support for the decoder. Also, this matrix is used for the LDPC encoder. In the following, the SPA is described.

➢ *Sum-Product algorithm*

The SPA performs the following operations [14]:

- Initialization of variable nodes

$\mu_{mn} = log\frac{Pr(v_n=1|c_n')}{Pr(v_n=0|c_n')}$, $m\epsilon\{1,\dots,M\}$,$n\epsilon\{1,\dots,N\}$(1)

- Iteration

- Parity check nodes computation

$$\beta_{mn} = 2\times\tanh^{-1}\left(\prod_{n'\in N_m/n}\tanh\left(\mu_{mn'}/2\right)\right) \quad (2)$$

- Variable nodes computation

$$\mu_{mn} = \gamma_n + \sum_{m'\in M_n/m}\beta_{m'n} \quad (3)$$

- A posteriori information

$$\tilde{\gamma} = \gamma + \sum_{m\in M_n}\beta_{mn} \quad (4)$$

- Decision

$$\hat{c}_n = \begin{cases}0 & si\tilde{\gamma}_n > 0\\ 1 & si\tilde{\gamma}_n < 0\end{cases} \quad (5)$$

Finally, the algorithm stops if the maximum number of iterations is reached or if the syndrome is zero.

III. EXACT LLR COMPUTATION

2^{2m}-QAM transmit, at each time, 2^{2m} binary symbols. Each set of $2m$ binary symbols is associated with a symbol

$c = a + jb$,where$and\ b\ \in$ $\{\pm 1,\pm 3,\pm 5,\dots,2m\pm 1\}$. After passing through the transmission channel, the observation relating to the symbol c is represented by the symbols $c' = a' + jb'$. At the reception, 2^{2m}-QAM-Gray demapping treat each symbols c' representative of the symbols c to extract $2m$ samples $\{\hat{u}_{n,i}\}, i\in\{1,\dots,2m\}$ each representative of a binary symbol $u_{n,i}$. The sample $\hat{u}_{n,i}$, the soft output demapping, is obtained using two relationships, $LLR(u_{n,i})$ (Log-Likelihood Ratio) or $APP(u_{n,i})$ (A Posteriori Probability). In this work, one used LLR computation. $LLR(u_{n,i})$, $i\in\{1,\dots,m\}$, is calculated as follows [15]:

$$LLR(u_{n,i}) = log\left[\frac{Pr\{(a_n',b_n')/u_{n,i}=1\}}{Pr\{(a_n',b_n')/u_{n,i}=0\}}\right] \quad (6)$$

Where $Pr\{(a_n',b_n')/u_{n,i}=w\}$ is the probability that the available couple is (a_n',b_n'); knowing the binary symbol $u_{n,i}$ is equal to w.

For a square constellation $m = 2p$, 2^{2p}-QAM has the particularity to be reduced to two amplitude modulations with 2^P states independently acting on two carriers in-phase and quadrature [11]. According to this property (the case of a square constellation):

➢ The p expressions in phase are consequently the following:

$$LLR(u_{n,i}) = log\left[\frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a_n'-a_{i,j}^0\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a_n'-a_{i,j}^1\right)^2\right\}}\right] i\in\{1,\dots,p\}(7)$$

With $a_{i,j}^k$ are possible values of the symbol a_n when the symbol $u_{n,i}$ to be transmitted has the value k (k = 0 or 1); $w = 0\ or\ 1$;

➢ The p relations in quadrature eventually lead to the following expressions:

$$LLR(u_{n,i}) = log\left[\frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b_n'-\alpha_n b_{i,j}^0\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b_n'-\alpha_n b_{i,j}^1\right)^2\right\}}\right] i\in\{p+1,\dots,2p\}(8)$$

With $b_{i,j}^k$ are possible values of the symbol b_n when the symbol $u_{n,i}$ to be transmitted has the value k (k = 0 or 1).

Equations (7) and (8) are the exact calculation of the LLR; it is the optimal calculation that represents the log-MAP algorithm [16-18]. However, it involves several operations. Several algorithms have been introduced to simplify the exact analysis of the LLR. In this work, we use two simplified algorithms: a max-log-MAP algorithm and a simplified max-log-MAP algorithm.

IV. SIMPLIFIEDLLR COMPUTATION (SIMPLIFIED MAX-LOG-MAPALGORITHM)

Using the simplified Max-Log-MAP algorithm, The LLR is simplified as follows [9]:

$$LLR(u_{n,i}) = \frac{\left(\min_{j\in\{1,\dots,2^{p-1}\}}\left(a_n'-a_{i,j}^0\right)\right)^2-\left(\min_{j\in\{1,\dots,2^{p-1}\}}\left(a_n'-a_{i,j}^1\right)\right)^2}{2\sigma^2}, \quad (9)$$

Where $i\in\{1,\dots,p\}$

And

$$LLR(u_{n,i}) = \frac{\left(\min_{j\in\{1,\dots,2^{p-1}\}}\left(b_n'-b_{i,j}^0\right)\right)^2-\left(\min_{j\in\{1,\dots,2^{p-1}\}}\left(b-b_{i,j}^1\right)\right)^2}{2\sigma^2}, \quad (10)$$

Where $i\in\{p+1,\dots,2\,p\}$

V. SIMULATION RESULTS

In this section, we present the effect of the LLR simplified calculation on the performance of a binary LDPC code, with a code rate of 1/2 and a parity check matrix of size 512 × 1024, and for a decoding algorithm using the LLR at its input: the SPA algorithm. The LDPC code is associated with two square constellations: 16-QAM and 64-QAM and the associated Gray coding, and a Gaussian channel.

Figures 1 and 2 show respectively for 16-QAM and 64-QAM, performance comparisons, on a Gaussian channel, between an LDPC code using the exact calculation of the LLR and an LDPC code the simplified analysis by applying the simplified max-log-MAP algorithm.

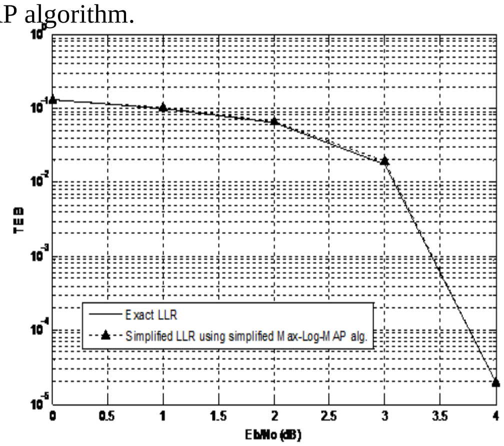

Fig. 1. Performance comparisons, under Gaussian channel, of (512, 1024) LDPC code using exact LLR computation and its simplified calculation using Simplified Max-Log-MAP algorithm, for 16-QAM

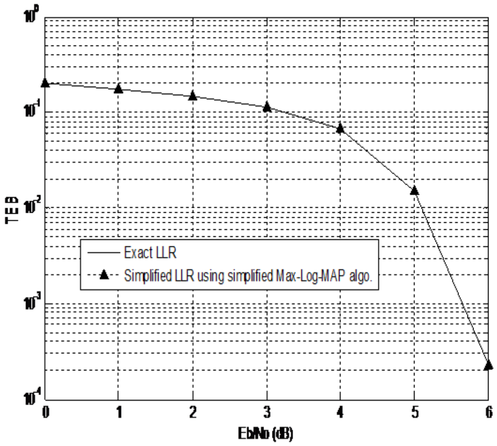

Fig. 2. Performance comparisons, under Gaussian channel, of (512, 1024) LDPC code using exact LLR computation and its simplified calculation using Simplified Max-Log-MAP algorithm, for 64-QAM

In figures 1 and 2, we can see that the LDPC code using the simplified LLR computations has a minimal performance loss for 16-QAM. For 64-QAM, there is no performance degradation. As a result, the simplification of LLR calculation can achieve good performance with a simple analysis.

VI. CONCLUSION

In this work, we have used binary LDPC codes to simplify the LLR using the simplified Max-Log-MAP algorithm. Simulation results show that the simplified max-log MAP algorithm is suitable for binary LDPC codes.

REFERENCES

[1] J. U. Duncombe, "*Infrared navigation—Part I: An assessment of feasibility," IEEE Trans. Electron Devices",* ED-11(1), pp. 34–39, Jan. 1959, 10.1109/TED.2016.2628402.

[2] Q. Wang, Q. Xie, Z.Wang, S. Chen and L. Hanzo, *"A Universal Low-Complexity Symbol-to-Bit Soft Demapper,"* IEEE Transactions on Vehicular Technology, 63(1), Jan. 2014, 119-130.

[3] L. Mostari, A. Bounoua and R. Meliani, *"Performance des turbo-codes utilisant la fonctiond'adaptation,"* Embedded System Conference, Algeria, May 05-06, 2009.

[4] L. Mostari, R. Meliani, A. Bounoua, *"Simplification de la fonction d'adaptation dans un système de transmission élaboré selon une approche pragmatique,"* International Conference on AppliedInformatics, Algeria, Nov. 15-17, 2009.

[5] R. Gallager, *"Low-Density Parity-Check Codes,"* IEEE Transaction Information Theory, 8(1), 1962, 21–28.

[6] R. G. Gallager, *"Low-density parity-check codes,"* Ph.D. dissertation, Department of Electrical Engineering, M.I.T., Cambridge, Mass., Juillet 1963.

[7] C. E. Shannon, *"A Mathematical Theory of Communication,"* Bell System Technical Journal, 27, Jul. Oct. 1948, pp. 379-423 and 623-656.

[8] L. Mostari, A. Bounoua, A. Taleb-Ahmed et R. Meliani, *"Decoding for binary LDPC Codes under Gaussian channel,"* The 4th Workshop on Codes, Cryptography and Communication System, Meknes, Morocco, Nov. 07-08, 2013.

[9] S. Y. Le Goff, *"Bit-Interleaved Turbo-Coded Modulations for Mobile Communications*", European Signal Processing Conference (EUSIPCO), Finlande, 2000.

[10] Xiang Liu and Martin Kosakowski, "*Max-Log-MAP Soft Demapper with Logarithmic Complexity for M-PAM Signals,"* IEEE Signal Processing Letters, 22(1), Jan. 2015, 50-53.

[11] C. Pan, J. Huang, K. Peng and L. Fan, *"Simplified Soft Demapping Algorithm for Gray-APSK,"* Wireless Communications, Networking and Mobile Computing (WiCOM), 2012 8th International Conference on, China, 2012.

[12] L. Mostari and A. Taleb-Ahmed, *"Simplified log-likelihood ratio for binary LDPC codes,"* Journal of Engineering Science and Technology Review, 07(1), Jan. 2017, 37-40.

[13] Tanner RM, "*A Recursive Approach to Low Complexity Codes,"* IEEE Transactions on Information Theory, IT-27(5), 1981, 533–547.

[14] S. J. Johnson, *"Low-Density Parity-Check Codes: Design and Decoding,"* Chapter in Wiley Encyclopedia of Telecommunications, 2002.

[15] M. Z. Alam, C. S. Islam and M. A. Sobhan, *"Using Log-Likelihood Relation for BER Analysis of QAM in Space Diversity,"* Journal of communications, 4(6), Jul. 2009, 371-379.

[16] F. Tosato and P. Bisaglia, *"Simplified Soft-Output Demapper for Binary Interleaved COFDM with Application to HIPERLAN/2,"* in IEEE International Conference Communication, 2, Aug. 2002, 664–668.

[17] K. Hyun and D. Yoon, "*Bit Metric Generation for Gray Coded QAM Signals,"* Proc. Inst. Elect. Eng., Commun., 152(6), Dec. 2005, 1134–1138.

[18] L. Wang, D. Xu, and X. Zhang, *"Low Complexity Bit Metric Generation for PAM Signals Based on Non-Linear Function,"* Electron. Lett., 17, Aug. 2011, 966–967.

[19] Rekha Rani. *"Analytical Performance of LDPC Codes with OC in the Presence of Interferers"*, International Journal of Engineering Trends and Technology (IJETT), V12(3),136-140 June 2014.

[20] Xiang Liu and Martin Kosakowski, *"Max-Log-MAP Soft Demapper with Logarithmic Complexity for M-PAM Signals,"* IEEE Signal Processing Letters, 22(1), Jan. 2015, pp. 50-53.