

Tuning of a Feedback Lag-Lead First-Order Compensator used with a Fractional Time Delay Double Integrating Process

Galal Ali Hassaan

*Emeritus Professor, Department of Mechanical Design & Production,
Faculty of Engineering, Cairo University, Giza, Egypt*

Abstract

The problem of controlling an unstable delayed double integrating process using a feedback first-order lag-lead compensator is studied. The effect of time delay of the process in a range between 0.1 and 0.8 seconds is considered. The compensator is tuned using MATLAB optimization toolbox with five forms of the objective function in terms of the error between the step time response of the closed-loop control system and the response steady-state value. Using the proposed compensator with the delayed double integrating process indicates the robustness of the compensator in the time delay range used with superior time-based specifications. The dynamics of the control system are compared with other technique based on using a feedforward first-order lag-lead compensator.

Keywords – *Feedback first-order lag-lead compensator, Fractional delayed double integrating process, Compensator tuning, Control system performance, Compensator robustness.*

I. INTRODUCTION

Position of the control compensator affects the dynamics of the closed-loop control system (stability, time based specifications and frequency based specifications). Delayed double integrating processes are examples of unstable processes which require extensive efforts in proper selection of suitable controllers or compensators and also looking for proper tuning techniques to achieve stable control system and accepted performance both in the time and frequency domains.

Zhang and Sun (1996) extended the Smith predictor to the general integrator/time delay process. They formulated a design procedure and developed simple tuning rules. They showed that there was a minimum-order compensator for the process [1]. O; Dwyer (2000) studied methods used to estimate process time delay based on time and frequency domain. He presented techniques for designing PD, PI and PID controllers for both SISO and MIMO process models in continuous and discrete time

domains. He studied also lead, lag and lead-lag controller structure. He presented structural and parameter optimized compensators using fuzzy logic, neural networks and expert systems [2]. Skogestad (2004) presented analytic rules for PID controller tuning. He modified the the rule for the integral term to improve disturbance rejection for integrating processes. He used a single tuning rule for a first-order or second-order time delay process [3]. Rao and Childambaram (2006) used a PID controller in series with a lead-lag compensator to control an open-loop unstable second-order plus time delay process. They used 2 tuning parameters and obtained significant improvement in load disturbance rejection performances [4].

Rao, Rao and Childambaram (2007) proposed a simple design of controllers for modified Smith predictor for unstable first-order plus time delay processes. There scheme had only one tuning parameter and they achieved robust control performance with significant improvement in the closed-loop performances [5]. Garcia and Albertos (2008) presented a dead-time compensator for stable and integrating processes when a reduced model of the process was considered. They derived a tuning procedure for better robustness and performance and studied the disturbance rejection performance [6]. Alcantara (2011) studied an analytical design of feedback compensators through linear control theory using a standard single-loop feedback configuration. His objectives were stability and robustness for set-point and disturbance rejection [7].

Nie, Wang, Wa, He and Qin (2011) considered a lead-lag compensator design problem for a class of unstable delay processes based on a new set of gain and phase margin specifications. They presented a tuning procedure and examples for illustration and comparison [8]. Hassaan, Al-Gamil and Lashin (2013) used a 4-parameters feedforward second-order lag-lead compensator to control a first-order with integrator process of 67.3 % maximum percentage overshoot and 12 s settling time. Through tuning the compensator they could reduce the

overshoot to 2.44 % and the settling time to 0.65 s [9].

Hassani, Tjahjowidodo and Do (2014) surveyed the various mathematical models of hysteresis in terms of their applications in modelling, control dynamic systems. They studied the control strategies in hysteric systems including feedforward control, feedback control and combination of feedback and feedforward [10]. Hassaan (2014) used a feedforward first-order lag-lead compensator to control a simple pole plus double integrator unstable process. He tune the compensator using an ISE objective function based on the gain and phase margins of the closed-loop control system. He compared his results with published work [11]. Hassaan (2015) investigated the robustness of feedback first-order lag-lead, feedforward second-order lag-lead and feedforward first-order lag-lead compensators used to control second-order processes against uncertainty in process parameters. He proved the robustness of the three compensators against process uncertainty [12].

II. PROCESS

The controlled process is delayed double integrating process having the transfer function, $G_p(s)$:

$$G_p(s) = (K_p/s^2) \exp(-T_d s) \quad (1)$$

Where K_p is the process gain and T_d is its time delay. It is dealt with the exponential term in Eq.1 through the first-order Taylor series as [13]:

$$\exp(-T_d s) \approx 1 - T_d s \quad (2)$$

Combining Eqs.1 and 2 gives the process transfer function as:

$$G_p(s) = (-K_p T_d s + K_p) / s^2 \quad (3)$$

The unit step response of the process using Eq.3 is shown in Fig.1.

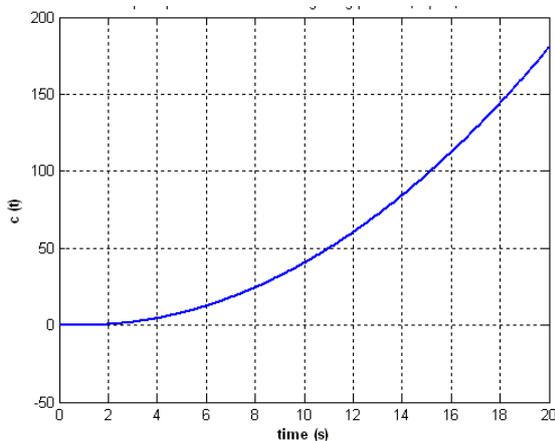


Fig.1 Step response of the double integrating process.

It is clear from Fig.1 that the double integrating process is an unstable 1. The compensator has to generate an stable feedback control system and also to achieve an accepted performance through tuning the compensator.

III. COMPENSATOR

The compensator used is a feedback first-order compensator having the transfer function, $G_c(s)$ [14, 15]:

$$G_c(s) = K_c(1 + T_z s) / (1 + T_p s) \quad (4)$$

Where K_c is the compensator gain, T_z and T_p are two time constants of the compensator.

IV. CONTROL SYSTEM TRANSFER FUNCTIONS

The block diagram of the feedback first-order lag-lead controlled system is shown in Fig.2.

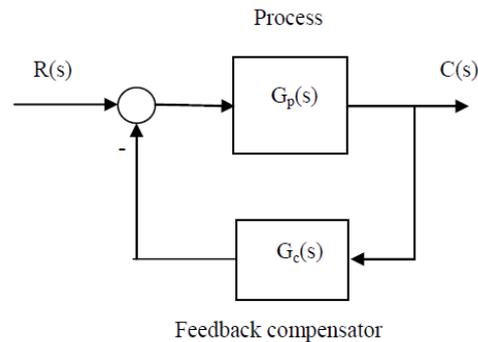


Fig.2 Closed-loop control system.

The open-loop transfer function of the control system incorporating the feedback first-order compensator and the delayed double integrating process, $G(s)H(s)$ is given by:

$$G(s)H(s) = (b_0 s^2 + b_1 s + b_2) / (T_p s^3 + s^2) \quad (5)$$

Where:

$$\begin{aligned} b_0 &= -K_c K_p T_z T_d \\ b_1 &= K_c K_p T_z - K_c K_p T_d \\ b_2 &= K_c K_p \end{aligned}$$

The closed-loop transfer function of the control system, $M(s)$ is given using the block diagram of Fig.2 by:

$$M(s) = (c_0 s^2 + c_1 s + K_p) / [T_p s^3 + (1+b_0)s^2 + b_1 s + b_2] \quad (6)$$

Where:

$$\begin{aligned} c_0 &= -T_p T_d K_p \\ c_1 &= T_p K_p - T_d K_p \end{aligned}$$

V. COMPENSATOR TUNING AND SYSTEM TIME RESPONSE

The compensator has to be tuned to achieve two purposes:

- (i) Providing a stable closed-loop control system.
- (ii) Controlling the performance of the closed-loop control system in terms of time-

based and frequency based specifications.

The compensator has three parameters: K_c , T_z and T_p . The compensator parameters are tuned as follows:

- For a unit process gain, a unit compensator gain will produce a zero steady-state error for a step reference input.
- Then, the remaining two compensator parameters T_z and T_p have to be tuned.
- The optimization toolbox of MATLAB is used for this purpose [16].
- The MATLAB command 'fminunc' is used [16].
- A number of objective functions based on the error between the step time response of the control system and its steady-state response are selected to tune the compensators. They are ITAE, ISE, IAE, ITSE and ISTSE [17-20].
- The tuning procedure is applied for a specific time delay of the double integrating process in the range $0.1 \leq T_d \leq 0.8$ s.
- The step response of the closed-loop control system is plotted using the command 'step' of MATLAB [21].
- The time-based specifications of the control system are extracted using the MATLAB command 'stepinfo' [21].
- The frequency-based specifications of the control system are extracted using the MATLAB command 'margin' [21].

A sample of the tuning results is shown in Table 1 for a 0.2 s time delay of the double integrating process and a unit gain.

Table 1: Compensator tuning for process unit gain and 0.2 s time delay.

Objective Function	K_c	T_z	T_p
ITAE	1	1.6772	0.2902
ISE	1	0.5688	0.0501
IAE	1	0.6716	0.0921
ITSE	1	1.0307	0.0801
ISTSE	1	1.2970	0.1275

The time response of the control system for a unit step input is shown in Figs.2 for time delay of 0.2 s for the five objective functions used in the compensator tuning

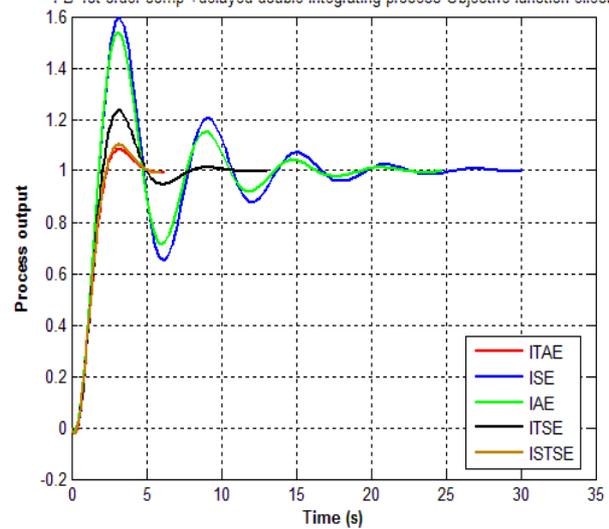


Fig.2 Control system time response for a 0.2 s time delayed double integrating process.

Varying the type of the optimization objective function has a remarkable effect on the time response of the control system. The best of them are ITAE and ISTSE providing minimum maximum overshoot. The worst of them is the ISE providing maximum percentage overshoot and settling time.

The effect of the time delay of the control system time response to a reference step input using the feedback first-order compensator is shown in Fig.3.

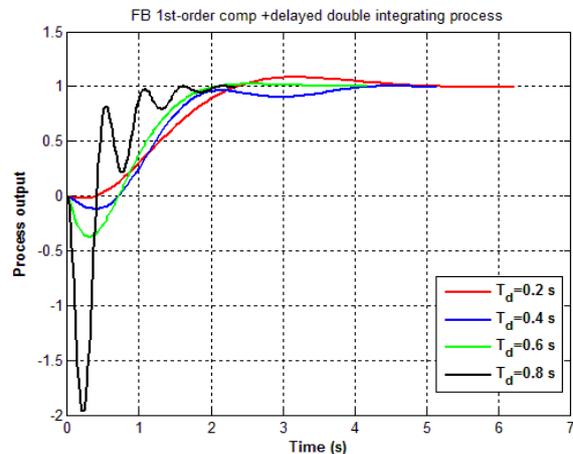


Fig.3 Effect of time delay on the step input time response using ITAE objective function.

The effect of the time delay of the double integrating process on some of the time-based specifications of the control system incorporating the feedback compensator is shown in Fig4.

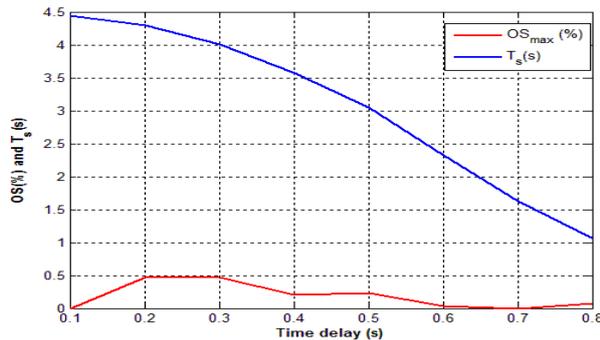


Fig.4 Effect of process time delay on overshoot and settling time.

The settling time decreases nonlinearly as the time delay increases. The maximum percentage overshoot changes between zero and 0.47 % during the time delay range between 0.1 and 0.8 s.

VI. COMPARISON WITH OTHER RESEARCH WORK

The unit time response of the control systems as presented in the present work using a first-order feedback compensator is compared with the work of the author for the same type of processes using a feedforward first-order lag-lead compensator [22]. The comparison is presented graphically in Fig.4.

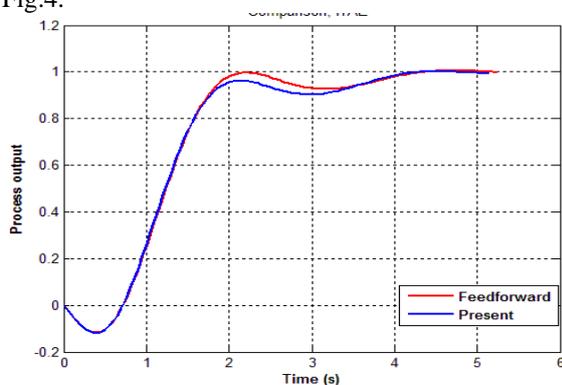


Fig.4 Time response comparison using ITAE objective function for 0.4 s time delay.

The time-based and frequency based specifications of the control system using the two control techniques are compared in Table 2.

Table 2: Time based specifications using two control techniques.

Control technique	Feedback first-Order compensator	Feedforward first-Order compensator
Maximum OS (%)	0.2100	0.5555
Maximum	11.8375	11.6324

US (%)		
Ts (s)	3.5687	3.5640
Peak time (s)	4.5014	3.1100

VII. CONCLUSION

- The dynamic problem of tuning a feedback lag-lead compensator for use with a delayed double integrating process was investigated.
- The effect of the process time delay on the tuning results and the time response of the control system was investigated.
- Five objective function forms were applied to tune the compensator.
- It has been shown that the applied objective function forms have remarkable effect on the tuning parameters of the compensator and the time response of the control to a set-point change and hence on the time-based specifications.
- The proposed compensator succeeded to produce a time response to a step reference input having a very small maximum percentage overshoot using ITAE objective function.
- Corresponding to the time delay range covered in the analysis, the maximum percentage overshoot did not exceed 0.408 %.
- The settling time of the control system step response did not exceed 4.436 seconds during the covered time delay range.
- The feedback first-order lag-lead compensator was robust against the change in the process time delay in the range between 0.1 and 0.8 seconds.
- Although the studied delayed double integrating process was an unstable one, the proposed compensator has provided good results when compared with the feedforward first-order compensator used with the same process.
- For a process of unit gain and 0.4 s time delay, the maximum percentage overshoot was 0.21 % for the present work compared with 0.555 % for the feedforward first-order compensator and the settling time was 3.5687 s compared with 3.5640 s for the feedforward compensator.
- The peak time of the control system step time response was 4.5014 s for the control system using feedback compensator compared with 4.6899 s using a feedforward compensator.

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